C and P symmetries

Charge conjugation (C):

Turns particle into anti-particle

electron e^- → e^+ positron

$B^0\ (\bar{b}d)$ → $\bar{B}^0\ (b\bar{d})$

And preserves helicity!

Parity reversal (P):

Inverts spatial coordinates

$r \rightarrow -r$

$p \rightarrow -p$

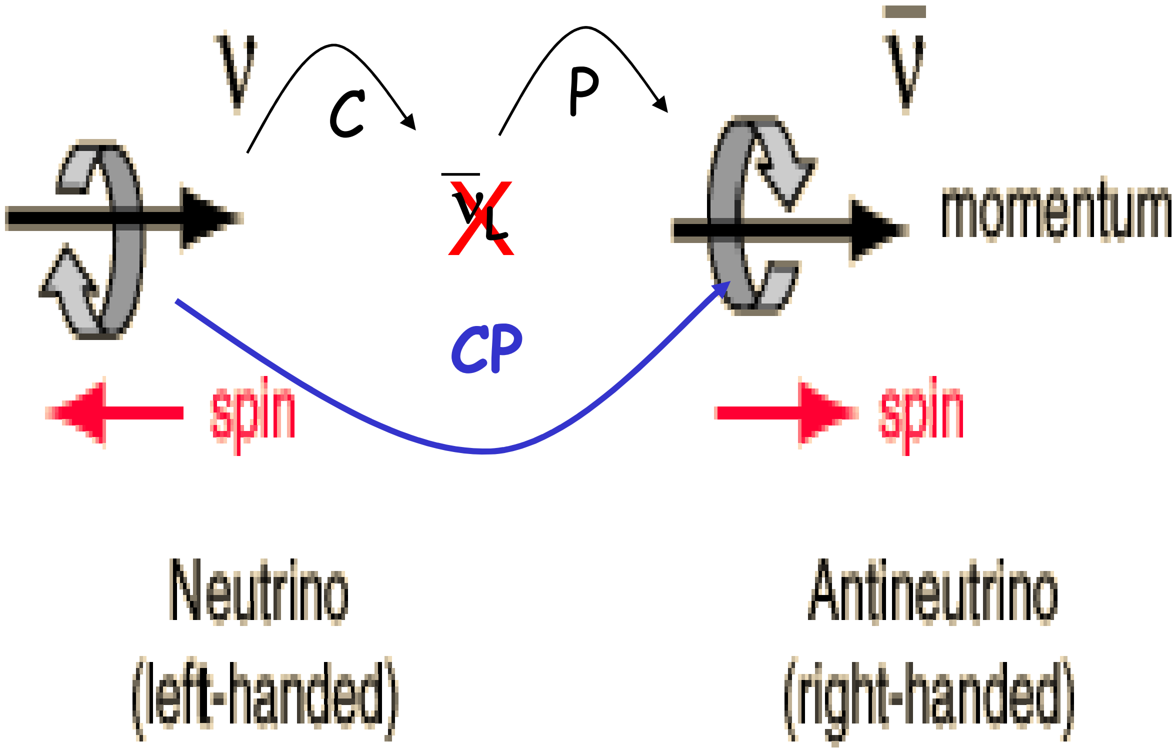
And reverses helicity

$e^-_L \rightarrow e^-_R$

Like looking in a mirror

C ,P, CP violation

Maximal violation of
C and P in weak
interactions



CP symmetry is also violated.
Small CPV effects discovered in the neutral Kaon system (1964)

Matter-AntiMatter Asymmetry

- At the Big Bang, matter and antimatter must have been created in equal quantities.
- The universe in which we live today consists primarily of matter.
- How come all the matter/antimatter did not annihilate?
- In order to achieve this asymmetry, 3 conditions must have occurred simultaneously at some early stage in the development of the universe.
  - Proposed by A. Sakharov in 1967.

Sakharov's conditions

1) There must have existed a baryon number violating process, which enabled the total B of the universe to change.
2) This process must violate both C and CP, so that the baryon violation affected matter and anti-matter differently.
3) The universe should have been out of thermal equilibrium at the time. This prevents the baryon and CP violating process proceeding backwards at the same rate.

CPV in the SM - I

- CPV was discovered in 1964 by Christenson et al. with the observation of the decay $K_L \to \pi^+\pi^-$
  - K^0 and $\overline{K^0}$ are produced in strong interactions and are connected by C
  - However, they are not eigenstates of CP

$$CP\left|K^0\right\rangle = \eta\left|\overline{K^0}\right\rangle$$

$$CP\left|\overline{K^0}\right\rangle = \eta'\left|K^0\right\rangle$$

  - Two linear combinations of K^0 and $\overline{K^0}$ are CP eigenstates:

$$\left|K_S\right\rangle = 1/\sqrt{2}\left(\left|K^0\right\rangle + \left|\overline{K^0}\right\rangle\right) \qquad CP = 1$$

$$\left|K_L\right\rangle = 1/\sqrt{2}\left(\left|K^0\right\rangle - \left|\overline{K^0}\right\rangle\right) \qquad CP = -1$$

  - Christenson et al. measured the ratio:

CP violating decay

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = (2.27 \pm 0.02) \times 10^{-3}$$

CP conserving decay

  - Most recent value (from NA48)

CPV in the SM - II

- Why looking for CP violation ?
  - Can look for CPV in the Standard Model, and baryon violation in GUTs.
  - CPV in the SM is too small (by orders of magnitude) to explain the matter/antimatter asymmetry in the universe.
  - One of the least experimentally tested areas of the Standard Model.
- Good place to look for new Physics!

P,C and CP: Conservation and Violation

- Symmetries lead to conservation laws.
  - A violated symmetry is a violated conservation law.
- Consider a process in which the initial state $|i\rangle$ interacts according to a Hamiltonian H resulting in a final state $\langle f|$.
  - These states may contain several particles
  - The interaction might be a decay.
- Consider a symmetry transformation U (which may be C, P or CP) which relates the states $|i\rangle$ $\langle f|$ to other well defined states $|i'\rangle$ $\langle f'|$
  - Eg $U|i\rangle = |i'\rangle$
  - If U=C, then $|i'\rangle$ would be the antiparticle(s) of $|i\rangle$

P,C and CP: Conservation and Violation

- If U is a symmetry of H we know:

$$[H,U]=0, H=U^{+}HU$$

- Hence

$$\langle f'|H|i'\rangle = \langle f|U^{+}HU|i\rangle = \langle f|H|i\rangle$$

- If U is the parity operator (P) then $< f\,|\,H\,|\,i >$ represents the Matrix Element for the process for given momenta and spin states and $< f'|\,H\,|\,i'>$ corresponds to states with opposite momenta and the same spins.

- If U is charge conjugation (C), the above means that the amplitude for a process and its anti-process is the same: e.g.

$$A(\pi^{+} \rightarrow \mu^{+}\upsilon_{\mu}) = A(\pi^{-} \rightarrow \mu^{-}\overline{\upsilon}_{\mu})$$

P,C and CP: Conservation and Violation

- If $|i(\vec{p},s)\rangle$ represents several particles with three momenta p and spins s:

$$\mathrm{P}: \quad \langle f(\vec{q},t)|H|i(\vec{p},s)\rangle = \langle f(-\vec{q},t)|H|i(-\vec{p},s)\rangle$$

$$\mathrm{C}: \quad \langle f(\vec{q},t)|H|i(\vec{p},s)\rangle = \langle \bar{f}(\vec{q},t)|H|\bar{i}(\vec{p},s)\rangle$$

$$\mathrm{CP}: \quad \langle f(\vec{q},t)|H|i(\vec{p},s)\rangle = \langle \bar{f}(-\vec{q},t)|H|\bar{i}(-\vec{p},s)\rangle$$

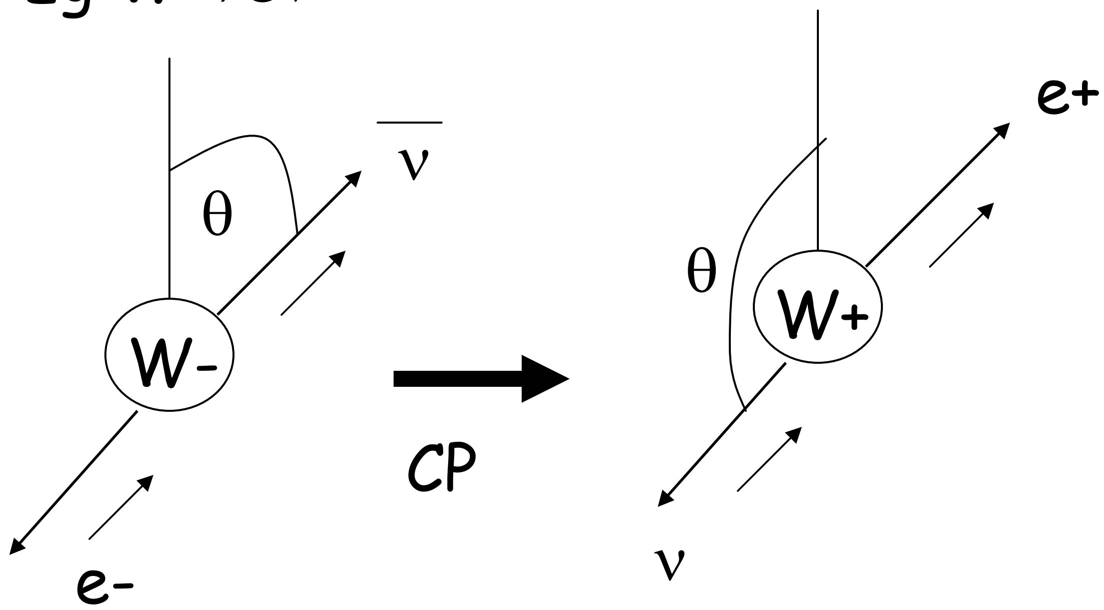
- In the case of CP conservation, when integrating over all final state configurations to find the total decay rate for a process, the total rates are equal for particle and antiparticle.
  - Even if the differential decay rates e.g. as a function of momentum are different.

P,C and CP: Conservation and Violation

- Hence:

  An asymmetry in the total decay rate for process and anti-process is a signal for CP as well as C violation.

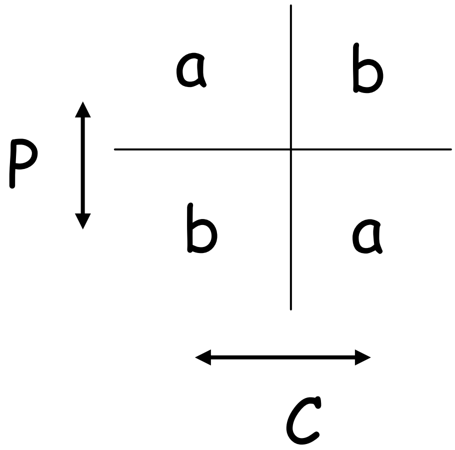
- Eg $W \to e\nu$



$$\frac{d\Gamma}{d\Omega} = \left|\frac{1+\cos\vartheta}{2}\right|^2 \qquad \frac{d\Gamma}{d\Omega} = \left|\frac{1-\cos\vartheta}{2}\right|^2$$

- Different angular distributions, but same TOTAL rate.
- CP conserving process.

CP Asymmetry



- Each quadrant represents the rate for a given sub-process, divided into parity and charge conjugate hemispheres.
- The P asymmetry is given by:

$$A_P = \frac{a-b}{a+b}$$

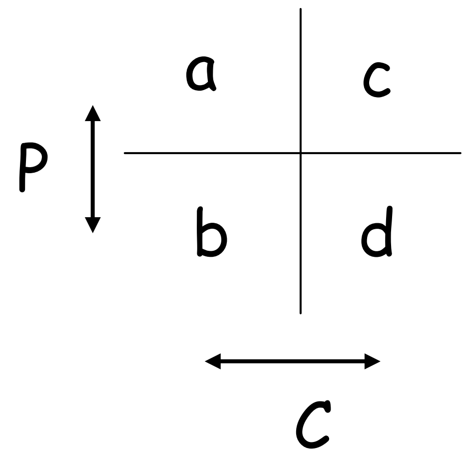
- The C asymmetry is given by:

$$A_c = \frac{a-b}{a+b}$$

- In this case C and P violation occur by the same amount and CP is conserved: $A_{CP} = 0$

CP Asymmetry

- One way to achieve CP violation is if the C and P violations are different.

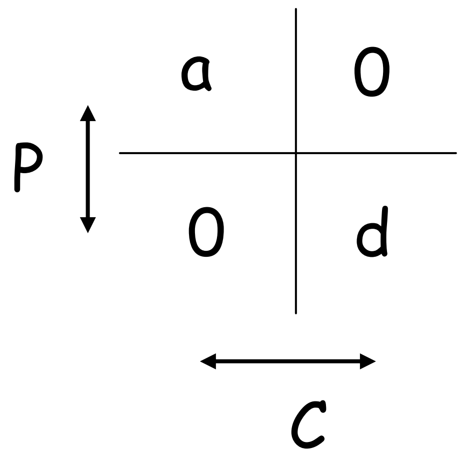


$$A_P = \frac{a-b}{a+b} \qquad A_c = \frac{b-d}{b+d} \qquad A_{CP} \neq 0$$

- This is NOT how CP violation arises in the Standard Model though.

CP Asymmetry

- In the SM, C and P are maximally violated.



$$A_P = \frac{a-0}{a+0} = 1 \qquad A_c = \frac{d-0}{d+0} = 1 \qquad A_{CP} \neq 0$$

- C and P violations occur by the same amount, but CP is still violated.

C and CP conservation

To summarize the consequences of C and CP violation on decay rates:

- C conservation: (independent of CP) $$\frac{d\Gamma(i \to f)}{d(\cos\vartheta)} = \frac{d\Gamma(\bar{i} \to \bar{f})}{d(\cos\vartheta)}$$
- CP conservation: (independent of C and P) $$\Gamma(i \to f) = \Gamma(\bar{i} \to \bar{f})$$
- If $\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})$ CP (and C) is violated

Time Reversal

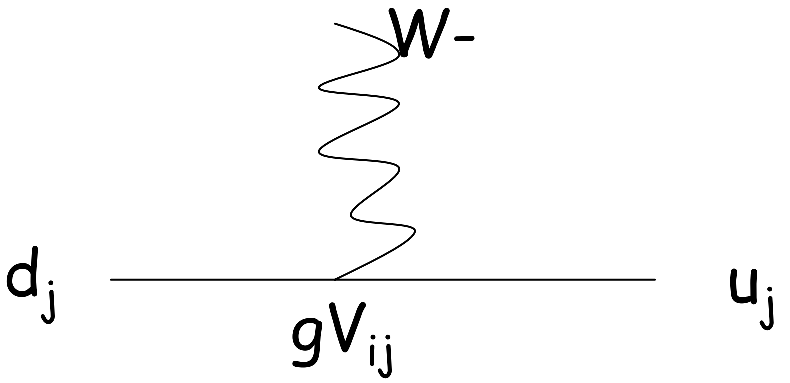
- The time-reversal operator T corresponds to reversing the direction (sign) of time in a theory.
- It is not a unitary operator.
  - It does not result in a conservation law.
  - There is no observable "T" quantum number.
- However, invariance under T can be tested as it results in constraints on the model.
- Under T:
  - Momenta and spin change sign.
  - Complex scalars (e.g. coupling constants) transform to their complex conjugate.
- It is believed that the strong and EM interactions conserve T, but the weak interaction does not.

CPT Theorem

- CPT is the combined transformation of C,P,T
  - Can be applied in any order as they commute.
- It can be proved (outside the scope of this course) that CPT is respected in any QFT which respects local and Lorentz invariance.
  - There are no known violations of CPT.
  - Some superstring theories lead to the possibility of CPT violation.
- The two main consequences of CPT symmetry are that the mass and lifetime of a particle are equal to those of the corresponding antiparticle.
- If a theory is invariant under CPT and violates CP(e.g. the SM) it also violates T.

The CKM Matrix

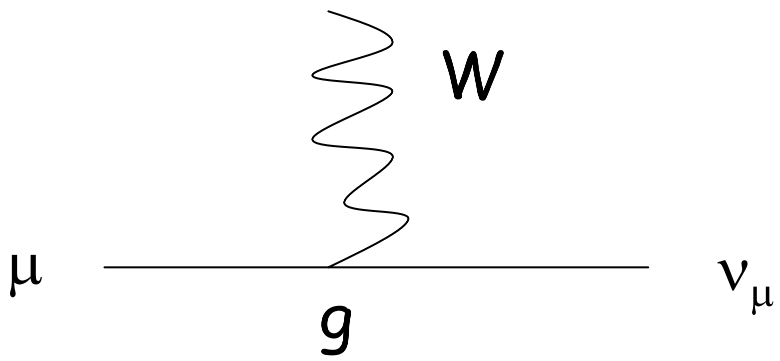
- The Cabibbo-Kobayashi-Maskawa Matrix parameterizes the couplings of the quarks to the $W^{+/-}$ boson in the charged current (with respect to the coupling of a μ and ν_μ to a W).



d_j is any quark of charge –1/3

u_i is any quark of charge 2/3

g is the gauge coupling constant.



The CKM Matrix

i,j=1: (u,d)   i,j=2: (c,s)   i,j=3: (b,t)

- There are 9 ways the above interaction can occur (ignoring antiparticles and time reversal interactions).
- Each of the 9 interactions has a different coupling.
- The coupling strengths are represented by the CKM matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Convention: write charge 2/3 quark first

The CKM Matrix

$d_j \rightarrow u_i + W^-$	V_{ij}
$\overline{d}_j \rightarrow \overline{u}_i + W^+$ (antiparticles)	V_{ij}^*
$u_i \rightarrow d_j + W^+$ (time reversal)	V_{ij}^*
$\overline{u}_i \rightarrow \overline{d}_j + W^-$	V_{ij} (from CPT)

- V_{ij} are in general complex.
  - True for all amplitudes in QM.

Origin of the CKM Matrix

- The CKM matrix arises from the gauge theory.
- The weak interaction ("Cabibbo rotated") states d′,s′,b′ are linear combinations of the mass eigenstates d,s,b:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- The states d′,s′,b′ each couple to the corresponding state u,c,t with strength g.

Origin of the CKM Matrix

- Hence there exist 6 doublets which couple to the W^- with the same coupling strength g:

$$\begin{pmatrix} d' \\ u \end{pmatrix}, \begin{pmatrix} s' \\ c \end{pmatrix}, \begin{pmatrix} b' \\ t \end{pmatrix}, \begin{pmatrix} e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\nu \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

- The CKM matrix represents a simple change of basis.
  - There are no cross-generational couplings e.g. s' and t do not couple via W exchange etc.
- The universality of the W.I. requires that
  - The states d',s',b' are orthogonal
  - The change of basis preserves their normalization (so that the overall coupling strengths remain equal in both bases).
- Hence the rows and columns of the CKM matrix are orthonormal.
  - The CKM matrix is unitary: $VV^+ = V^+V = I$
- This is the only condition on the matrix which is required by the SM.
  - Showing that the matrix is not unitary would be a clear indication of new physics.

Physical Phases in V

- An nxn *unitary* matrix has n^2 independent real parameters.
- An nxn *orthogonal* matrix has n(n-1)/2 independent real parameters (rotation angles).
- Hence an nxn *unitary* matrix can be parameterized by:
  - n(n-1)/2 rotation angles
  - n^2 – [n(n-1)/2]=n(n+1)/2 phases
- However, observables are all of the form:

$$\langle u_i | V_{ij} | d_j \rangle$$

  - Each quark wave function can be changed by an arbitrary phase without changing any observable quantity.
  - For n generations, there are 2n such phases (n for the up-type quarks, and n for the down-type quarks).

Physical Phases in V

- A common phase change of all the quark spinors will not change V at all, so it is only the relative phases which matter.
  - There are 2n-1 relative phases.
  - Hence the number of independent physical phases in V is:

$$\frac{n(n+1)}{2}-(2n-1)=\frac{(n-1)(n-2)}{2}$$

- For n=2, V can be parameterized with 1 rotation angle and no phases:

$$\begin{pmatrix} d' \\ s' \end{pmatrix}=\begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix}$$

- For n=3, V can can be parameterized with 3 rotation angles and 1 phase.
- For n=4, V can can be parameterized with 6 rotation angles and 3 phases.
- *It is the phase which is responsible for CP violation in the SM.*