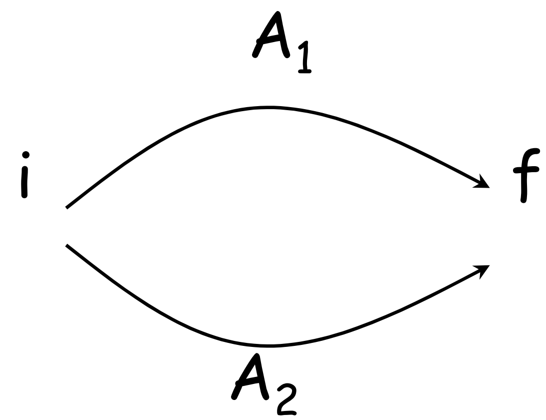
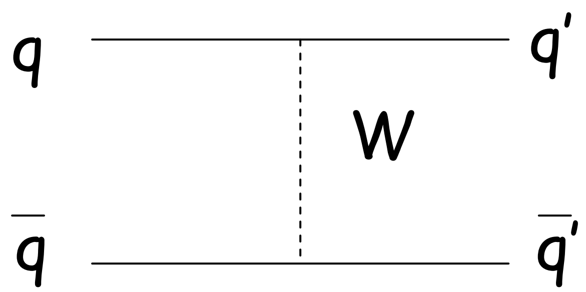
Requirements for CP Violation

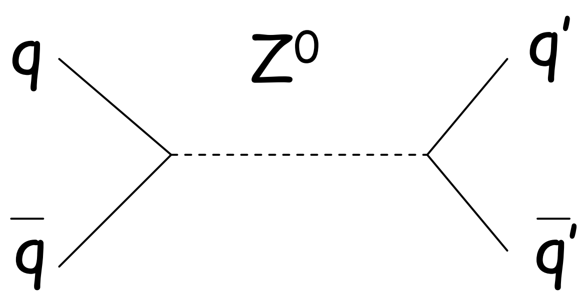
- (1) We need a process which can occur via at least two diagrams. These diagrams must interfere with each other.

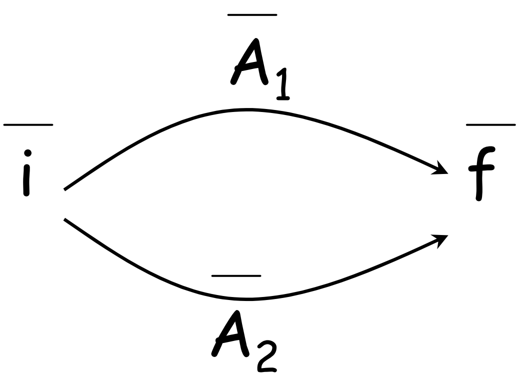


$A(i \rightarrow f) = A_1 + A_2$

Eg:







$\bar{A} = \bar{A}_1 + \bar{A}_2$

CP conjugate

Requirements for CP Violation

- (2) Each of the two amplitudes needs to be made of two factors,

  a_i and α_i (i=1,2,...)

  which transform under CP as follows:

$$a_i \rightarrow a_i \qquad \alpha_i \rightarrow \alpha_i^* \quad \Rightarrow \quad A_i = a_i \alpha_i \qquad \bar{A}_i = a_i \alpha_i^*$$

and a_i and α_i must have at least one non-zero relative phase, i.e.:

$$Arg(a_i) \neq Arg(a_j)$$

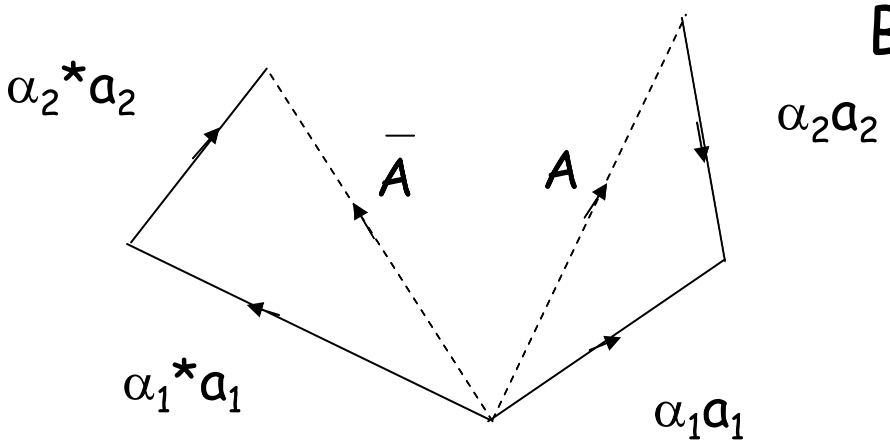
$$Arg(\alpha_i) \neq Arg(\alpha_j)$$

CP Asymmetry

- We can then write for A and $\bar{A}$:

$$A = A_1 + A_2 = \alpha_1 \mathrm{a}_1 + \alpha_2 \mathrm{a}_2$$
$$\bar{A} = \bar{A}_1 + \bar{A}_2 = \alpha_1^* \mathrm{a}_1 + \alpha_2^* \mathrm{a}_2$$

- In the complex plane:



By construction:

$$|A|^2 \neq |\bar{A}|^2$$

- Define a CP asymmetry:

$$A_{CP} = \frac{\Gamma(i \to f) - \Gamma(\bar{i} \to \bar{f})}{\Gamma(i \to f) + \Gamma(\bar{i} \to \bar{f})} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}$$

CP Asymmetry

- Since:

$$|A|^2 = A \bullet A^* = (\alpha_1 a_1 + \alpha_2 a_2)(\alpha_1^* a_1^* + \alpha_2^* a_2^*)$$

$$|\overline{A}|^2 = \overline{A} \bullet \overline{A}^* = (\alpha_1^* a_1 + \alpha_2^* a_2)(\alpha_1 a_1^* + \alpha_2 a_2^*)$$

after some algebra we obtain:

$$A_{CP} = \frac{4\,\mathrm{Im}(\alpha_1 \alpha_2 *)\,\mathrm{Im}(a_1 a_2 *)}{2|\alpha_1 *|^2 |a_1|^2 + 2|\alpha_2 *|^2 |a_2|^2 + 4\,\mathrm{Re}(\alpha_1 \alpha_2 *)\,\mathrm{Re}(a_1 a_2 *)}$$

$$\Downarrow$$

$$A_{CP} \propto \mathrm{Im}(\alpha_1 \alpha_2^*)\,\mathrm{Im}(a_1 a_2^*)$$

- So A_{CP} is proportional to the phase difference between a_1 a_2 and α_1, α_2

$$A_{CP} \propto \sin[Arg(\alpha_1) - Arg(\alpha_2)] \cdot \sin[Arg(a_1) - Arg(a_2)]$$

Neutral B mesons

- In experiments we produce neutral B mesons of definite flavour:

$$B^0 \equiv \bar{b}\, d; \bar{B}^0 \equiv b\bar{d}$$

- These are not the states of definite mass (denoted B_L and B_H)
- They are not necessarily the states of definite CP (denoted B_+ and B_-)

Neutral B meson mixing

- Let's consider an arbitrarily b flavored state: $a|B^0> + b|\overline{B}^0>$
- Its evolution with time will be governed by:

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H \begin{pmatrix} a \\ b \end{pmatrix} \qquad H = M - \frac{i}{2}\Gamma$$

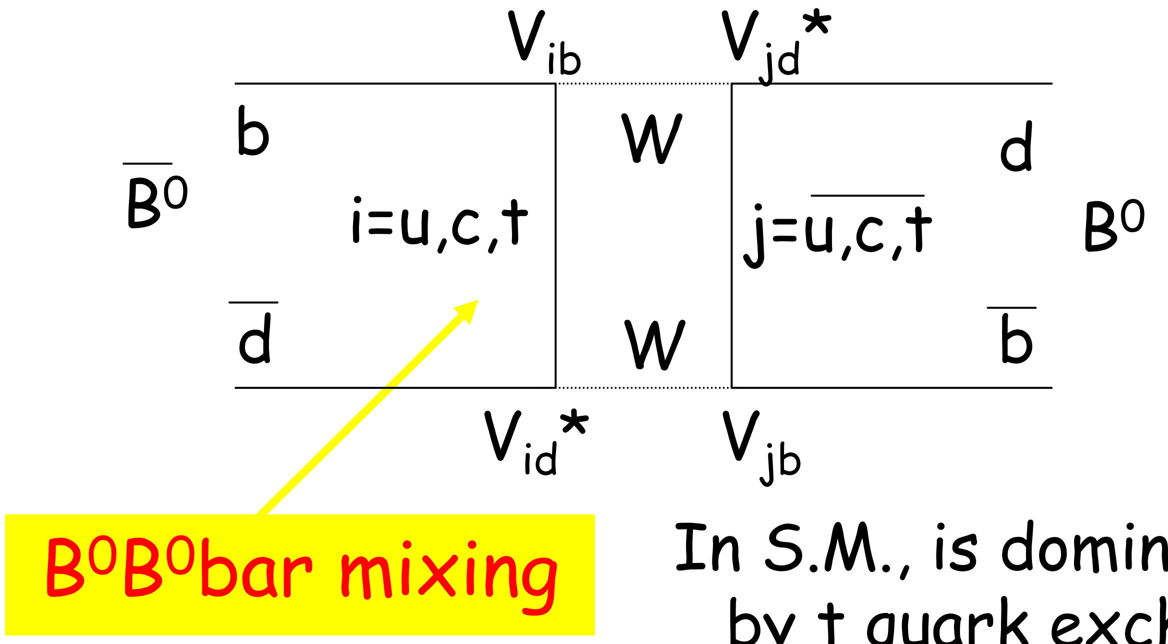
M and Γ are 2x2 Hermitian matrices.

- The Mass Matrix is due to processes with virtual intermediate states which do not lead to the decay of the meson.
- The decay matrix (Γ) is due to processes with real intermediate states which can lead to the decay of the meson.

Interpretation of M and Γ matrices

- Both M and Γ are Hermitian matrices:
$$M_{21} = M_{12}^*$$
$$\Gamma_{21} = \Gamma_{12}^*$$
- B^0 and B^0bar are particle and antiparticle. Conservation of CPT means:
$$M_{11} = M_{22} = M$$
$$\Gamma_{11} = \Gamma_{22} = \Gamma$$
- Diagonal elements of M arise from quark masses and binding forces.
- Off diagonal elements of M are due to B^0-B^0bar transitions with virtual intermediate states (see next slide)

Interpretation of M and Γ matrices



In S.M., is dominated by t quark exchange

- Diagonal elements of Γ arise from decays like (X is a real state): $B^0 \to X; \overline{B}^0 \to \overline{X}$

- Off-diagonal elements of Γ arise from decays like:

$$\overline{B}^0 \to X \to B^0$$

Some formalism of neutral B meson mixing

- Diagonalising H gives two eigenvectors, the states with definite mass and lifetime:

$$|B_L> = p|B^0> + q|\overline{B}^0>$$

$$|B_H> = p|B^0> - q|\overline{B}^0>$$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}; p^2 + q^2 = 1$$

- The eigenvalues corresponding to these two states are:

$$m_{L,H} - i\frac{\Gamma_{L,H}}{2}$$

where $m_{L,H} \equiv M \pm \mathrm{Re}(Q)$ and $\Gamma_{L,H} \equiv \Gamma \pm \mathrm{Re}(Q)$

$$\text{and } Q = \sqrt{(M_{12}^* - \frac{1}{2}\Gamma_{12}^*)(M_{12} - \frac{1}{2}\Gamma_{12})}$$

Some formalism of neutral B meson mixing

- We define:

$$\Delta\Gamma = \Gamma_H - \Gamma_L; \qquad \Delta m = m_H - m_L$$

- It can be shown that (e.g. look in the BaBar physics book):

$$\Delta\Gamma << \Delta m$$

- With some algebra:

$$\Delta\Gamma = \frac{2\,\mathrm{Re}(M_{12}M_{12}{}^*)}{|M_{12}|}; \qquad \Delta m = 2|M_{12}|$$

- In summary: $i\frac{d}{dt}B_{L,H} = (m_{L,H} - \frac{1}{2}\Gamma_{L,H})B_{L,H}$

The solutions are:

$$|B_{L,H}(t)> = \exp[-i(m_{L,H} - i\frac{\Gamma_{L,H}}{2})t]\,|B_{L,H}(0)>$$

Time evolution

- We are interested in the time evolution of the physical B meson states, B^0 and B^0bar.
- These are an admixture of the mass eigenstates B_L and B_H
- A state which is initially a pure B^0 state evolves with time according to:

$$|B^0{}_{phys}(t)> = f_+(t)|B^0> + \frac{q}{p} f_-(t)|\overline{B}^0>$$

where
$$f_+(t) \equiv e^{-imt} e^{-\Gamma t/2} \cos\frac{\Delta m t}{2}$$

$$f_-(t) \equiv e^{-imt} e^{-\Gamma t/2} i \sin\frac{\Delta m t}{2}$$

Time evolution

- m and Γ are the mean mass and decay width:

$$m = \frac{m_L + m_H}{2}; \Gamma = \frac{\Gamma_L + \Gamma_H}{2}$$

- The probability of observing a B^0 at time t will be:

$$< B^0 \mid B^0{}_{phys}(t) >$$

- Similarly, an initially pure B^0bar state evolves with time as:

$$| \overline{B}^0{}_{phys}(t) >= \frac{p}{q} f_-(t) | B^0 > + f_+(t) | \overline{B}^0 >$$