How many forms of CPV?

- Direct CP violation:

$$\boxed{N(i \rightarrow f) \neq \overline{N}(\bar{i} \rightarrow \bar{f})}$$

- Indirect CP violation:

$$\boxed{\Gamma(B^0 \rightarrow \overline{B}^0) \neq \Gamma(\overline{B}^0 \rightarrow B^0)}$$

- Interference of decay with and without mixing:

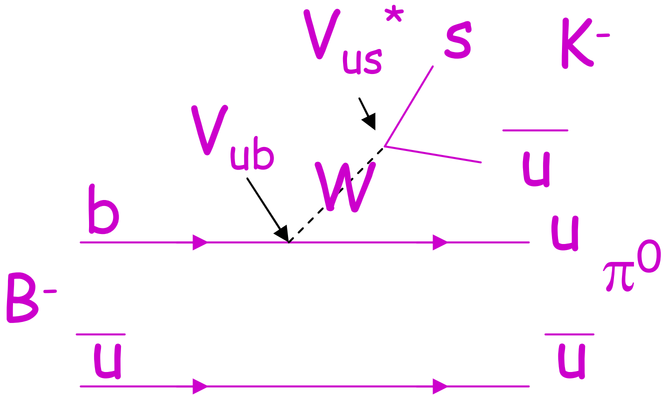
$$\begin{tikzcd} & B^0\text{bar} \arrow[dr] & \\ B^0 \arrow[ur] \arrow[rr] & & f \end{tikzcd}$$

$$\neq$$

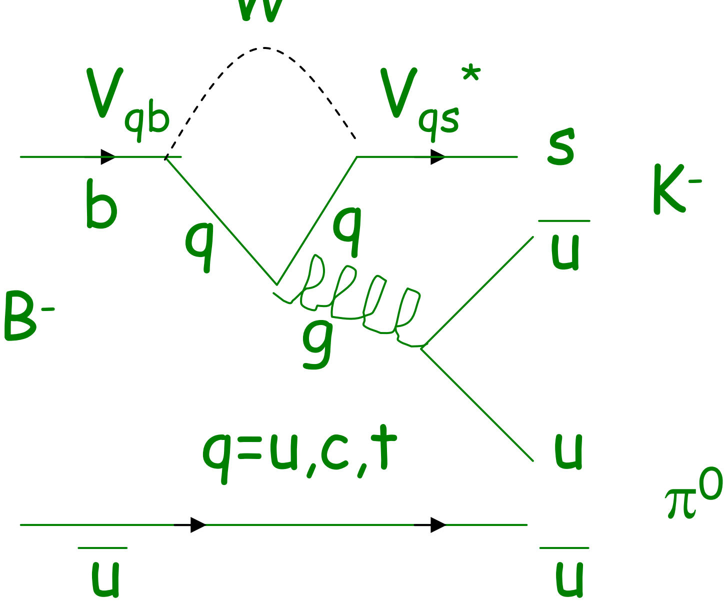
$$\begin{tikzcd} & B^0 \arrow[dr] & \\ \overline{B^0} \arrow[ur] \arrow[rr] & & f \end{tikzcd}$$

Direct CP Violation

- This is the simplest form of CP violation.
- It involves the interference between two decay diagrams.
- Example: $B^- \to K^- \pi^0$



"Spectator"



"Penguin"

Direct CP Violation

- The diagram with q=t dominates since $|V_{tb}|$~1
- We can represent the amplitudes for the two decays as:

$$A = ae^{i\alpha}V_{ub}V_{us}* \qquad B = be^{i\beta}V_{tb}V_{ts}*$$

amplitude

Strong phase

CKM M.E. contains weak phase

- For the CP conjugate decay: $B^+ \to K^+\pi^0$

$$\bar{A} = ae^{i\alpha}V_{ub}*V_{us} \qquad \bar{B} = be^{i\beta}V_{tb}*V_{ts}$$

$$\Gamma - \bar{\Gamma} = (A+B)^2 - (\bar{A}+\bar{B})^2$$

$$= 4ab\sin(\alpha-\beta)\,\mathrm{Im}(V_{ub}V_{us}*V_{tb}*V_{ts})$$

Direct CP Violation

- In the Wolfenstein Parameterisation:

$$A_{CP} = \frac{\Gamma - \overline{\Gamma}}{\Gamma + \overline{\Gamma}}$$

$$= \frac{2ab\eta \sin(\alpha - \beta)}{a^2 \lambda^2 (\rho^2 + \eta^2) + b^2 \lambda^{-2} + 2ab\rho \cos(\alpha - \beta)}$$

- In order to achieve a significant asymmetry we need:
  - Comparable magnitudes for the two amplitudes
  - Different strong interaction phases
  - Different weak phases.
- There are several similar modes:

$$B^- \to K, K^* + \pi, \rho, \eta$$

Direct CP Violation

- Branching ratios for these modes is $O(10^{-5})$ but A_{CP} may be as large as 10%.
- Such decay modes are experimentally easy to study
  - Just have to count number of decays of particle/anti-particle.
- Decays are self-tagging
  - Sign of K gives sign of B
- Not trivial to relate individual elements to CKM M.E.
  - Need to know amplitudes a,b
  - Need to know strong phase difference $\alpha-\beta$.

Direct CP Violation

- For the K^0 system, it can be shown that in the Standard Model due to penguins:

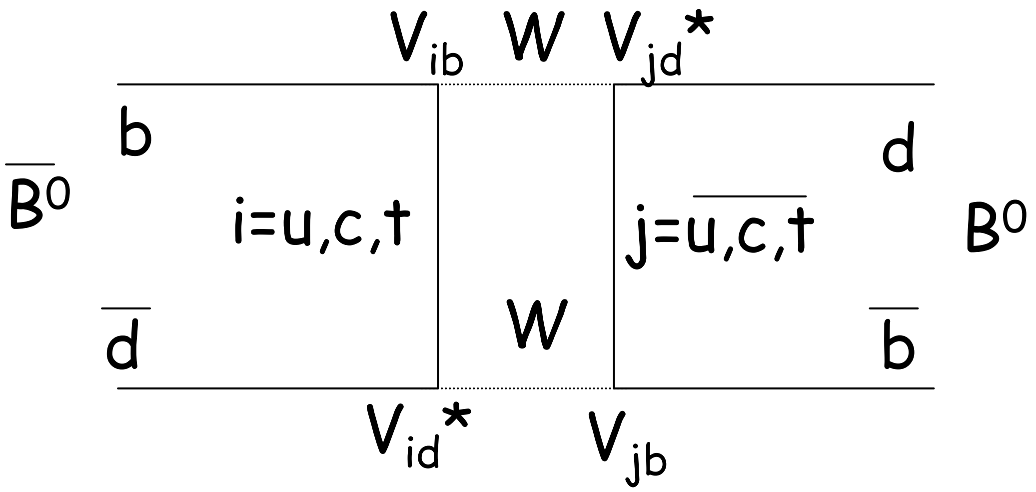
$$\frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_L^0 \to 3\pi)} \neq \frac{A(K_L^0 \to \pi^0\pi^0)}{A(K_L^0 \to 3\pi)}$$

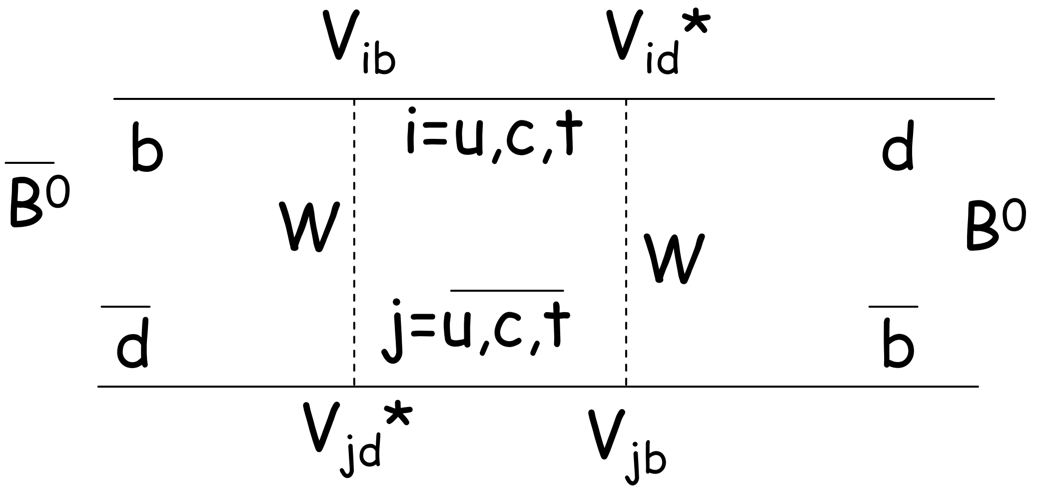
- The difference between the two quantities is parameterised by a quantity ε′/ε.
  - A non-zero value would provide evidence for direct CP violation.
- Direct CP violation in K^0 decay was first confirmed experimentally about 2 years ago by KTeV and NA48.
  - New direct evidence from BaBar:

$$\frac{N(B^0 \to K^+\pi^-) - N(\overline{B^0} \to K^-\pi^+)}{N(B^0 \to K^+\pi^-) + N(\overline{B^0} \to K^-\pi^+)} = -0.133 \pm 0.030 \pm 0.009$$

Indirect CP Violation

- A neutral meson (eg B^0 or K^0) can spontaneously tranform into its own anti-particle via the second order weak interaction:





Indirect CP Violation

- 18 diagrams in total.
- They can interfere with each other.
- Total amplitude is:

$$A = \sum_{i,j} A_{ij} \lambda_i \lambda_j$$

$$\text{where } \lambda_i = V_{ib} V_{id} * \text{etc}$$

- By Unitarity: $\sum_i \lambda_i = 0$

- Under CP transformation:

$$A_{ij} = A_{ij}$$

$$\lambda_{i,j} = \lambda_{i,j} *$$

Indirect CP Violation

- Hence this process satisfies the general requirement for CP violation and:

$$|A|^2 \neq |\overline{A}|^2$$

- The rate for a B^0 to transform to a B^0bar is different than for a B^0bar to transform to a B^0.

- We can study this type of CP violation using di-lepton events.
  - In B-factories, B^0 and B^0bar are always produced in pair:

$$\boxed{\begin{array}{l} B^0 \rightarrow l^+ X \nu_l \\ \overline{B}^0 \rightarrow B^0 \rightarrow l^+ X \nu_l \end{array}}$$

Indirect CP Violation

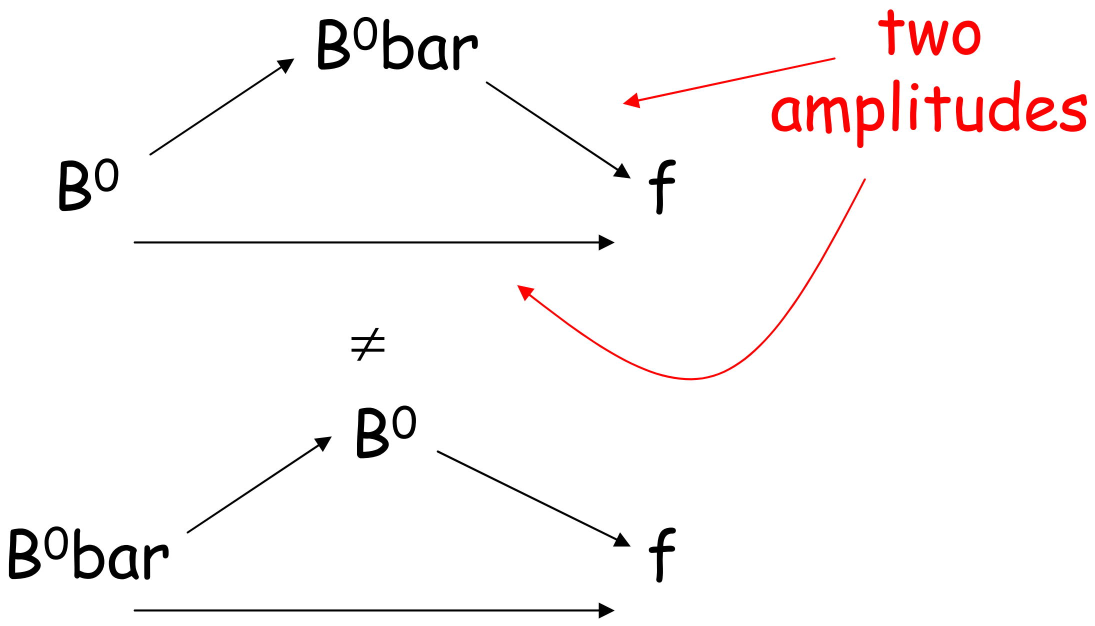
- If we produce pairs of B^0-B^0bar mesons, we can investigate indirect CP violation from the asymmetry:

$$A = \frac{N_{++} - N_{--}}{N_{++} + N_{--}}$$

- The size of the asymmetry is predicted to be O(0.001).
  - Need ~100 million ll pairs to see this effect
- CLEO II: A < 0.18 at 90% CL.
- High luminosity B factories.

Interference in decays with and without mixing

- Consider the decay to a state which is accessible from both B^0 and B^0bar decays:



- The path via mixing can interfere with the direct decay.

- It is especially easy to calculate in the S.M if f is a CP eigenstate:

$$|\bar{f}>=CP|f>=\pm f$$

- It is best when the direct CP violation in the decay step is negligible.
  - Such decays are always Cabibbo-suppressed with BF ~ 10^{-3} to 10^{-6}.

- Consider the amplitude for a time evolved B^0 or B^0bar to decay to a CP eigenstate f_{CP}:

$$<f_{CP}|\overline{B}^0{}_{phys}(t)>=\frac{p}{q}f_-(t)<f_{CP}|B^0>+f_+(t)<f_{CP}|\overline{B}^0>$$

$$<f_{CP}|B^0{}_{phys}(t)>=f_+(t)<f_{CP}|B^0>+\frac{q}{p}f_-(t)<f_{CP}|\overline{B}^0>$$

- Define: $\lambda \equiv \frac{q}{p} \frac{< f_{CP} \mid \overline{B}^0 >}{< f_{CP} \mid B^0 >}$

- The time-dependent rate for a initially pure B^0 to decay to f_{CP} is:

$$\Gamma\left(B^0{}_{phys}(t) \rightarrow f_{CP}\right) = \left|< f_{CP} \mid B^0 >\right|^2 e^{-\Gamma t}$$

$$\times \left\{ \cos^2 \frac{\Delta m t}{2} + |\lambda|^2 \sin^2 \frac{\Delta m t}{2} - 2\,\mathrm{Im}\,\lambda \sin \frac{\Delta m t}{2} \cos \frac{\Delta m t}{2} \right\}$$

- Similarly, for an initially pure B^0bar state:

$$\Gamma\left(\overline{B}^0{}_{phys}(t) \rightarrow f_{CP}\right) = \left|< f_{CP} \mid \overline{B}^0 >\right|^2 e^{-\Gamma t}$$

$$\times \left\{ \cos^2 \frac{\Delta m t}{2} + \frac{1}{|\lambda|^2} \sin^2 \frac{\Delta m t}{2} - 2\,\mathrm{Im}\,\lambda^{-1} \sin \frac{\Delta m t}{2} \cos \frac{\Delta m t}{2} \right\}$$

- To a good approximation:

$$\frac{q}{p} = \sqrt{\frac{M_{12}^{*} - i\Gamma_{12}^{*}/2}{M_{12} - i\Gamma_{12}/2}} \approx \sqrt{\frac{M_{12}^{*}}{M_{12}}} \quad \text{and} \quad \left|\frac{q}{p}\right| = 1 \quad (|\Gamma_{12}| << |M_{12}|)$$

- If a single combination of mixing ME contributes to the B^0 and B^{0}bar decay (eg a single weak decay diagram dominates):

$$\left|\frac{< f_{CP} \mid \overline{B}^{0} >}{< f_{CP} \mid B^{0} >}\right| = 1$$

$$|\lambda| = \left|\frac{q}{p}\right| \left|\frac{< f_{CP} \mid \overline{B}^{0} >}{< f_{CP} \mid B^{0} >}\right| = 1$$

And λ is a pure phase: $\lambda = e^{i\phi}$

- It is possible to show that:

$$\Gamma\left(B^0{}_{phys}(t) \to f_{CP}\right) = \left|< f_{CP} \mid B^0 >\right|^2 e^{-\Gamma t}\{1 - \operatorname{Im}\lambda \sin\Delta mt\}$$

$$\Gamma\left(\overline{B}^0{}_{phys}(t) \to f_{CP}\right) = \left|< f_{CP} \mid B^0 >\right|^2 e^{-\Gamma t}\{1 + \operatorname{Im}\lambda \sin\Delta mt\}$$

- The rate for a particle which started as a pure B^0 is different for one which started as a pure B^0bar provided:
  - The B^0 and B^0bar mix so Δm is non-zero.
  - Im λ is non-zero – possible as the weak decay can involve a physical phase.

- When direct CP violation is negligible: Im(λ) = sin($2\theta_{(\text{unit. Triangle})}$)

$$\begin{cases} B \to \pi^+\pi^- \text{ measures } \sin 2\alpha \\ B \to J/\psi K_s^0 \text{ measures } \sin 2\beta \end{cases}$$

- Time dependent asymmetry:

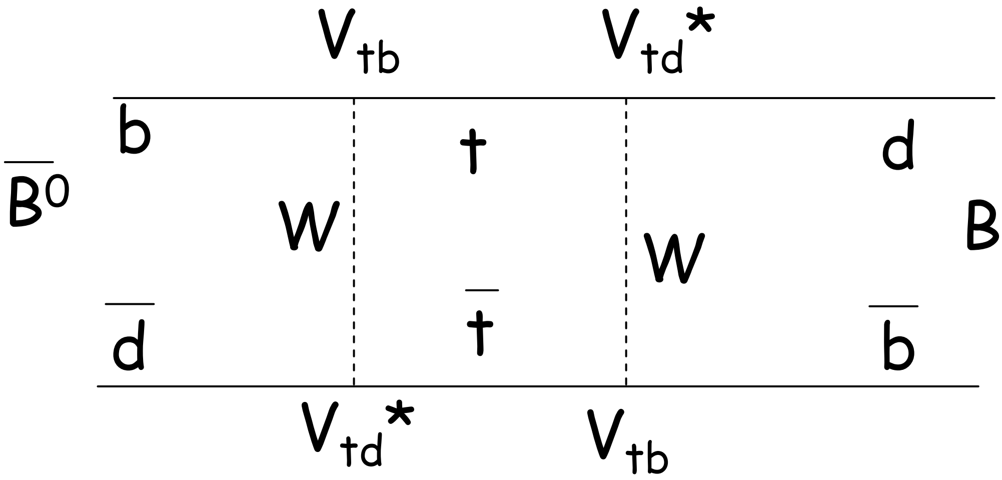
$$A_{CP}(t) = \frac{\Gamma\left(\overline{B}^0{}_{phys}(t) \to f_{CP}\right) - \Gamma\left(B^0{}_{phys}(t) \to f_{CP}\right)}{\Gamma\left(\overline{B}^0{}_{phys}(t) \to f_{CP}\right) + \Gamma\left(B^0{}_{phys}(t) \to f_{CP}\right)}$$

$$= \operatorname{Im}\lambda \sin \Delta m t$$

where:

$$\lambda \equiv \frac{q}{p}\frac{< f_{CP} \mid \overline{B}^0 >}{< f_{CP} \mid B^0 >} \approx \sqrt{\frac{M_{12}{}^*}{M_{12}}}\frac{< f_{CP} \mid \overline{B}^0 >}{< f_{CP} \mid B^0 >}$$

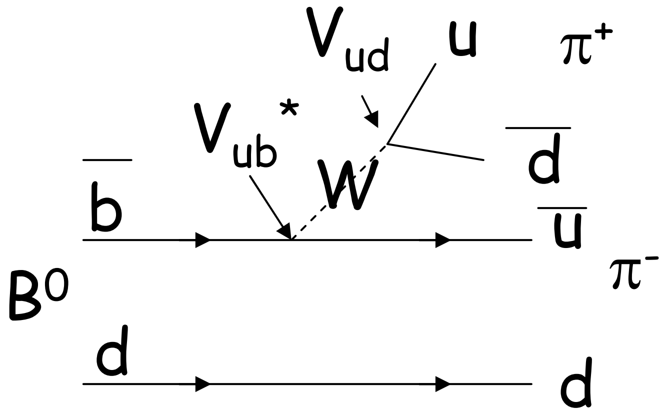
- For $B_d^0 - \overline{B}_d^0$ mixing, the process is dominated by t quark exchange:



$$\boxed{M_{12} \propto \left(V_{td}{}^* V_{tb}\right)^2}$$

The Decay $B^0 \to \pi^+\pi^-$

- Now we can consider some specific decays.



- From the Feynman diagram:

$$< f\,|B^0 > \propto V_{ub}^{\ *} V_{ud}$$

- Hence:

$$\lambda \approx \sqrt{\frac{M_{12}^{\ *}}{M_{12}}}\frac{< f_{CP}\,|\,\overline{B}^0 >}{< f_{CP}\,|\,B^0 >} = \frac{V_{tb}^{\ *}V_{td}}{V_{tb}V_{td}^{\ *}}\frac{V_{ub}V_{ud}^{\ *}}{V_{ub}^{\ *}V_{ud}}$$

The Decay $B^0 \to \pi^+\pi^-$

- In the standard parameterisation:

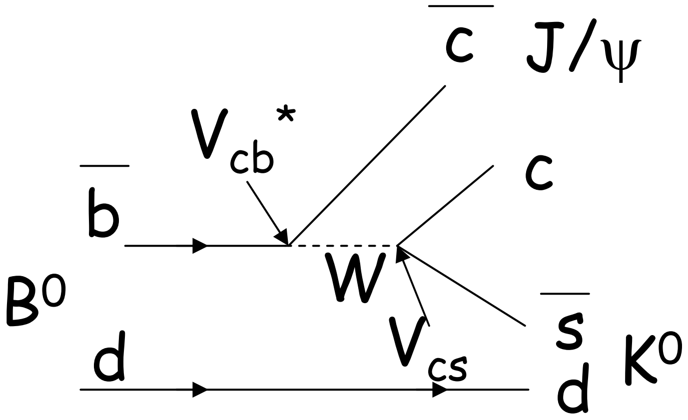
$$V_{tb} \approx 1; V_{ud} \approx 1 \Longrightarrow \mathrm{Im}\,\lambda \approx \mathrm{Im}\left(\frac{V_{td}}{V_{td}^*}\frac{V_{ub}}{V_{ub}^*}\right) = \sin 2\alpha$$

$$\Downarrow$$

$$\boxed{A_{CP}(t) = \mathrm{Im}\,\lambda \sin \Delta mt = \sin 2\alpha \sin \Delta mt}$$

- In practice, the penguin contribution in this decay is not negligible.
  - There are more than one diagram contributing to the decay.
  - This makes the asymmetry more difficult to interpret in terms of sin2α.

The Decay $B^0 \rightarrow J/\psi K^0_S$



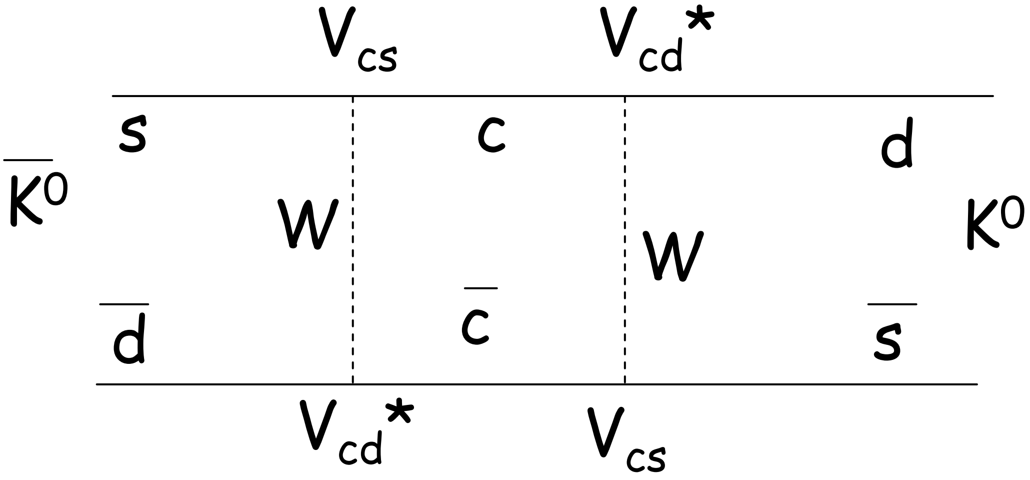
- From the Feynman diagram:

$$< f \middle| B^0 > \propto V_{cb}^{\,*} V_{cs}$$

- The final state is only a CP eigenstate to the extent that the neutral K^0 mixes to produce physical states K^0_s and K^0_L which are (approximately) CP eigenstates.

The Decay $B^0 \rightarrow J/\psi K^0_S$

- We assume K^0-K^0bar mixing is dominated by the box diagram:



- Therefore we must include in λ a factor which arises from this:

$$\lambda = \frac{q}{p}\frac{< f_{CP} \,|\, \overline{B}^0 >}{< f_{CP} \,|\, B^0 >}\left(\frac{p}{q}\right)_K$$

$$\left(\frac{p}{q}\right)_K = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}$$

The Decay $B^0 \to J/\psi K^0_S$

- Hence putting it all together:

$$\lambda = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}$$

$$= \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}$$

- In the standard parameterisation:

$$V_{tb} \approx 1; \mathrm{Im}(V_{cb} V_{cd}^*) \approx 0$$

$$\mathrm{Im}\,\lambda \approx \mathrm{Im}\left(\frac{V_{td}}{V_{td}^*}\right) = -\sin 2\beta$$

$$\boxed{A_{CP}(t) = -\sin 2\beta \sin \Delta m t}$$

(For the decay $B^0 \to J/\psi K^0_L$ the asymmetry has the same magnitude but with the opposite sign)

Measuring the angles

- For B_d mesons:

Process	Example modes	Angle
b→c cbar s	B0→J/ψK0s, B0→ψ(2s)K0s, B0→χ K0s, B0→J/ψK0L,...	β
b→c cbar d	B0→D+D-,...	β
b→c ubar d b→u cbar d	B0→DK	γ (very hard)
b→ubar ud	B0→π+π-, B0→ρ0π0, B0→π0π0, B0→ω0π0,....	α (hard)

- And many more!

Measuring the angles

- γ is best measured using B_s mesons eg:

$$B_s \rightarrow J/\psi\phi$$

- Would require the B factories to operate at the Υ(5S) resonance instead of the Υ(4s) resonance:
  - Upgrade the machine which costs $$$$!

Beyond the Standard Model

- The predictions we have made use the following assumptions:
    - $\Delta\Gamma \ll \Delta m$
    - Direct decay of $B \to f_{CP}$ is dominated by a single combination of quark mixing ME.
    - BBbar and KKbar mixing are dominated by a single box diagram.
    - The CKM matrix is unitary.
- Models beyond the SM do not necessarily have all the same properties.

Beyond the Standard Model

- Therefore, we need to measure the angles in as many different ways as possible
- Check for consistency between different measurements of the same angle, and for consistency between all angles and sides.

NEED B FACTORIES !