ORIGIN OF CALIBRATION CONSTANTS IN THE CAVITY BEAM POSITION MONITOR SYSTEM AT ATF2

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1. Electronics Processing

The monopole signal produced in the reference cavity has a frequency $omega_R$, is independent of beam offset and is used to account for charge and bunch length. It is assumed that the signal is at a maximum at time $t = 0$ and decays exponentially with a decay time τ_R.

$$V_R(t) = R_a \exp\left[-\frac{t}{2\tau_R}\right] \cos(\omega_R t) \tag{1}$$

The processing shall be split into three parts: transmission to the mixer ($T1$), the rest of the electronics (E) and transmission to the digitiser ($T2$). Each of the two transmission sections has a specific length Δx and an impedance that determines the wave number k. The wave number in $T1$ will be very different from in $T2$ because the frequency upstream of the mixer is much larger. The electronics are simplified to a single impedance and an effective length. Therefore, there is a phase advance between the cavity and the digitiser given by

$$r_T = k_R^{T1}\Delta x_R^{T1} + k_R^E \Delta x_R^E + k_R^{T2}\Delta x_R^{T2} \tag{2}$$

where the superscript R refers to the reference signal and the superscripts denote the hardware section. The mixer uses a local oscillator of amplitude R_{LO} and phase r_{LO} relative to the reference signal to mix down to the intermediate frequency $\omega_R - \omega_{LO}$. The total gain/attenuation between the cavity and the digitiser arises from the gain of the amplifier and mixer in the electronics, the attenuation arising from the cable losses and any added attenuation. These have been combined in a single factor R_g. The signal at the digitiser is therefore given by

$$V_R^d(t) = R_a R_{LO} R_g \exp\left[-\frac{t}{2\tau_R^d}\right] \cos((\omega_R - \omega_{LO})t + r_T - r_{LO}) \tag{3}$$

The decay constant has also changed to τ_R^d because of the filters in the electronics.

The dipole signal from the position cavity is linearly dependent on the beam offset x as well as the bunch tilt α and the beam angle θ. The signal also peaks at a time $t + \Delta t_{beam}$ that accounts for the difference in the time at which the beam arrives at each cavity.

$$V_P(t \geq \Delta t_{beam}) = \exp\left[-\frac{t - \Delta t_{beam}}{2\tau_P}\right] [A_x x \cos(\omega_P(t - \Delta t_{beam})) + (A_\alpha \alpha - A_\theta \theta)\sin(\omega_P(t - \Delta t_{beam}))] \tag{4}$$

$$V_P(t \geq \Delta t_{beam}) = \exp\left[-\frac{t - \Delta t_{beam}}{2\tau_P}\right] \sqrt{(A_x x)^2 + (A_\alpha \alpha - A_\theta \theta)^2} \cos\left(\omega_P(t - \Delta t_{beam}) - \tan^{-1}\left(\frac{A_\alpha \alpha - A_\theta \theta}{A_x x}\right)\right) \tag{5}$$

This signal goes through similar processing but the constants are different because of the frequency differences of the two cavities, the length differences of the cables and manufacturing differences of the electronics. Furthermore, the local oscillator used for mixing down to the intermediate frequency will be at a different amplitude and phase after being transmitted between the two cavities. Amplitude factors and phase changes are therefore written as P and p instead of R and r respectively. The position cavity signal at the digitiser is given by Eq. 6 where the square root in Eq. 5 is replaced by P_a and the arctangent term, that is the phase change due to the tilt signal, by Θ.

(6)

$$V_P^d(t) = P_a P_{LO} P_g \exp\left[-\frac{t-\Delta t_{beam}}{2\tau_P^d}\right] \cos((\omega_P - \omega_{LO})t - \omega_P \Delta t_{beam} + p_T - p_{LO} + \Theta)$$

2. Digital Processing

The digitiser sampling window opens at a time t_0 before the arrival of the position or reference signal. The window start time is adjusted to take Δt_{beam} into account so that t_0 is the same for both position and reference signals. This has the effect of boosting the position signal such that $t \rightarrow t + \Delta t_{beam}$ which swaps the phase change of $\omega_P \Delta t_{beam}$ to $\omega_{LO} \Delta t_{beam}$ Both signals are then mixed down to zero frequency using a complex oscillator that starts at the beginning of the sampling window $t - t_0$ at the same frequency as the digitised signal. The high frequency component from the mixing is then removed using a digital filter that again, changes the amplitude by a factor R_f and the decay constant to τ^0.

(7)

$$V_R^0(t) = \frac{R_a R_{LO} R_g R_f}{2} \exp\left[-\frac{t}{2\tau_R^0}\right] \exp\left[i((\omega_R - \omega_{LO})t_0 + r_T - r_{LO})\right]$$

(8)

$$V_P^0(t) = \frac{P_a P_{LO} P_g P_f}{2} \exp\left[-\frac{t}{2\tau_P^0}\right] \exp\left[i((\omega_P - \omega_{LO})t_0 - \omega_{LO}\Delta t_{beam} + p_T - p_{LO} + \Theta)\right].$$

Both signals are then sampled at a time t_s and the position signal is divided by the reference signal and the real and imaginary parts are

(9)

$$I = \frac{P_a P_{LO} P_g P_f}{R_a R_{LO} R_g R_f} \exp\left[\frac{t_s}{2\tau_R^0} - \frac{t_s}{2\tau_P^0}\right] \cos((\omega_P - \omega_R)t_0 - \omega_{LO}\Delta t_{beam} + p_T - r_T + r_{LO} - p_{LO} + \Theta)$$

(10)

$$Q = \frac{P_a P_{LO} P_g P_f}{R_a R_{LO} R_g R_f} \exp\left[\frac{t_s}{2\tau_R^0} - \frac{t_s}{2\tau_P^0}\right] \sin((\omega_P - \omega_R)t_0 - \omega_{LO}\Delta t_{beam} + p_T - r_T + r_{LO} - p_{LO} + \Theta)$$

The IQ rotation angle is given by

(11) $$\theta_{IQ} = (\omega_P - \omega_R)t_0 - \omega_{LO}\Delta t_{beam} + p_T - r_T + r_{LO} - p_{LO}.$$

This rotates the real and imaginary signals to pure position and angle signals. The position signal I' is

(12)

$$I' = I\cos(\theta_{IQ}) + Q\sin(\theta_{IQ})$$

(13)

$$I' = \frac{P_a P_{LO} P_g P_f}{R_a R_{LO} R_g R_f} \exp\left[\frac{t_s}{2\tau_R^0} - \frac{t_s}{2\tau_P^0}\right] (\cos(\theta_{IQ} + \Theta)\cos(\theta_{IQ}) + \sin(\theta_{IQ} + \Theta)\sin(\theta_{IQ}))$$

(14)

$$I' = \frac{P_a P_{LO} P_g P_f}{R_a R_{LO} R_g R_f} \exp\left[\frac{t_s}{2\tau_R^0} - \frac{t_s}{2\tau_P^0}\right] \cos\Theta$$

This is equivalent to substituting $t = -\Delta t_{beam}$ into Eq. 5 and multiplying by a constant.

$$I' = \frac{P_{LO} P_g P_f}{R_a R_{LO} R_g R_f} \exp\left[\frac{t_s}{2\tau_R^0} - \frac{t_s}{2\tau_P^0} - \frac{\Delta t_{beam}}{2\tau_P}\right] V_P(-\Delta t_{beam}) \tag{15}$$

and since Eq. 5 is equivalent to Eq. 4,

$$I' = \frac{P_{LO} P_g P_f}{R_a R_{LO} R_g R_f} \exp\left[\frac{t_s}{2\tau_R^0} - \frac{t_s}{2\tau_P^0}\right] A_x x. \tag{16}$$

The position scale is therefore given by

$$S = \left[\frac{P_{LO} P_g P_f}{R_a R_{LO} R_g R_f} \exp\left[\frac{t_s}{2\tau_R^0} - \frac{t_s}{2\tau_P^0}\right] A_x\right]^{-1} \tag{17}$$

where

$$x = SI'. \tag{18}$$

3. Conclusion

From Eq. 11 and Eq. 17, it is possible to see all the sources of the calibration constants and how they might vary. Besides transmission of electronics, the IQ rotation angle will vary with timing changes. Any change in the trigger timing Δt_0 will alter the IQ rotation angle by $(\omega_P - \omega_R)\Delta t_0$. Any change in the frequency of the two cavities will also affect the IQ rotation angle through this term. Δt_{beam} can be the same order of magnitude as t_0 but should not cause changes in phase from pulse to pulse barring changes in the frequency of the local oscillator $\omega_L O$ since it is almost completely accounted for in the digitiser window spacing. Δt_{beam} should be relatively stable since the beam is relativistic and is moving in longitudinally much faster than it is transversely so it's path length will not change by a great deal. It is possible to account for trigger timing variation in the analysis. Frequency changes are possible to measure by phase flattening but this is not possible on a pulse by pulse basis.

Phase changes in transmission may occur with changes in impedance. The effect would be smaller for the C-band cavities than for the S-band cavities since the electronics are much closer to the cavities and so there is less time before down-mixing when the wavenumber is highest and so the phase change is greatest. Changes in the phase of the local oscillator should show up in both r_{LO} and p_{LO} and the two effects should cancel. However, impedance changes in cables could affect the local oscillator at the position cavity only.

The electronic sources that affect the position scale are mostly measurable in a system of the size at ATF2. However, long term stability may be an issue. The effect of the filters on the decay constants is not so trivial. However, it is not expected to be a large effect and should be similar for each cavity and therefore, predictable.