PH3010: Cluster Galaxies

Joseph Bayley
Dr John Hargreaves

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Abstract

This projects was too observe the Hydra I cluster so that its mass and radius could be determined. By finding the density profile of the galaxy cluster it is possible to determine where the edge of the galaxy is i.e its radius. The model of an isothermal gas sphere subject to a central force was used to fit to the observed data. The scaling factors gave a radius of the cluster as (1.24 ± 0.03) Mpc and a central density of (6.81 ± 0.5) $Gal/arcmin^2$. This radius could then be used alongside the Virial theorem to find the mass of the cluster, this was found to $(1.26 \pm 3) \cdot 10^{44} kg$ or $(6.6 \pm 1.5) \cdot 10^{14} M\odot$. By assuming all galaxies are similar it was possible to calculate the average density of the universe to $(2.7 \pm 0.6) \cdot 10^{-27} kg \cdot m^{-3}$ which compared to the critical density of the universe, $(7.9 \pm) \cdot 10^{-27} kg \cdot m^{-3}$, is within the same order of magnitude.

1 Introduction

The Hydra I cluster is a cluster of galaxies in the Hydra constellation located around 67Mpc from our galaxy. By observing the distribution of galaxies in the cluster it is possible to find the density profile of the cluster. Initially the background number of stars will need to be found so that they can be taken away from the counts in the galaxy. By finding this background count it also allows the calculation of the limiting magnitude in the count area. This is then used alongside the distance modulus to find the lowest absolute magnitude in the image.
The density profile of the cluster can be found by counting the number of galaxies in concentric circles around the center of the galaxy and dividing by the area. This density profile will fall to zero at some point once the background count has been removed. This point is at the edge of the cluster, therefore also the radius of the galaxy cluster.
By using the theory of an isothermal gas sphere subject to a central force it is possible to find a dimensionless differential equation of the Lane-Emden form. This can then be solved to find the density profile of the gas sphere. As the measured density profile is a projection on the visible plane the isothermal gas sphere model needs to be projected on to a plane. This projection can then be fitted to the observed data to find more accurate measurements for the radius of the cluster.
By using the Virial theorem, equating the kinetic and potential energies, it is possible to get an expression for the mass of the cluster, using the radius of the cluster and the given velocities of individual galaxies.
This calculated mass can be then used to calculate the average density of matter in the universe assuming that galaxy clusters are separated by around 50Mpc.

2 Experiment

2.1 Data collection

To collect the data an image from the UK Schmidt telescope in Australia analysed. Firstly the larger galaxies were marked onto an overlay. Then to distinguish between the smaller galaxies a magnified glass was used, the stars could be seen to have spikes and were very clear however the galaxies had a blur around the image. This overlay was then scanned into the computer so that a python program could be written to count the number of galaxies in concentric circles. The scale of the image was kept by marking two points on the overlay 20cm apart then the scale could be defined.

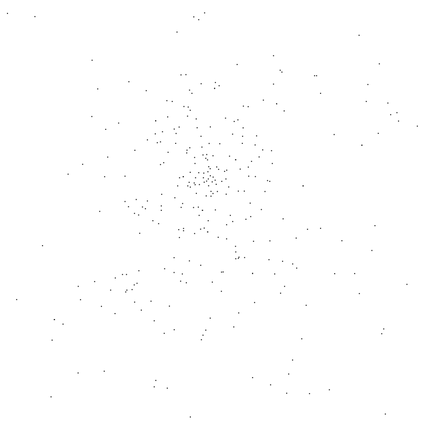

Figure 1: Marked galaxies in Hydra Cluster

2.2 Data Analysis

Initially the number of background galaxies needed to be calculated in the image so that a more accurate reading could be found for the cluster. This was found by taking a number of 2cm square areas outside the boundary of the cluster ,counting the galaxies inside and diving by the area. The background count was:

$$N_{back} = (0.83 \pm 0.1) counts \cdot cm^{-2}$$

where N is the number of galaxies counted. To find the magnitude of the faintest galaxy in the image the count per square degree was found by using the plate scale

$$1cm = 11.2 arcmins \tag{1}$$

by inserting this number count into the equation:

$$logN = 0.6m + C \tag{2}$$

where N is the number of galaxies per square degree, m is the limiting magnitude and C is constant that can be found to around $C = -9$. This then gives a limiting magnitude of:

$$m = (16.08 \pm 0.5)$$

From this limiting magnitude it is possible to calculate the absolute magnitude of the faintest galaxy in the image, by using the distance modulus.

$$m - M = 5logd - 5 \tag{3}$$

where the distance to the Hydra cluster is known to be $67Mpc$. The absolute magnitude was found to be:

$$M = (-18.05 \pm 0.5)$$

The center of the cluster was initially estimated then a program was used to find the center of mass of the points. Concentric circles are placed around this center at radii of intervals 0.5cm, as Figure 2 shows.

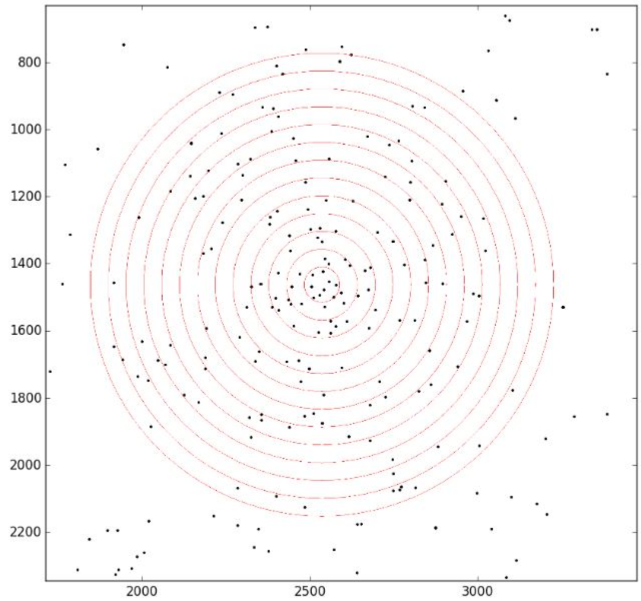

Figure 2: Marked galaxies in Hydra Cluster with concentric circles at 0.5cm intervals

The area of the concentric circles was calculated with:

$$A = 2\pi \left(\frac{r_1 + r_2}{2} \right) dr \tag{4}$$

By dividing the number count within each area by the area the densities could be found for different radii of the cluster. This density profile of the Hydra cluster could then be plotted, as shown in Figure 3.

By looking at where the density profile falls to zero, it is clear that this is where the edge of the galaxy cluster is. Also as the distance to the cluster is known as 67Mpc it is possible from basic trigonometry to find the radius in Mpc.

$$67[Mpc] \cdot \tan(r[rad]) = r[Mpc] \tag{5}$$

The radii were measured to:

$$R_O = (63.7 \pm 0.5)\ arcmins = (1.24 \pm 0.03)\ Mpc$$

The inner radius of the cluster can be found by seeing where the density falls to half the central density, this was measured to:

$$R_I = (9.82 \pm 0.5)\ arcmins = (0.19 \pm 0.03)\ Mpc$$

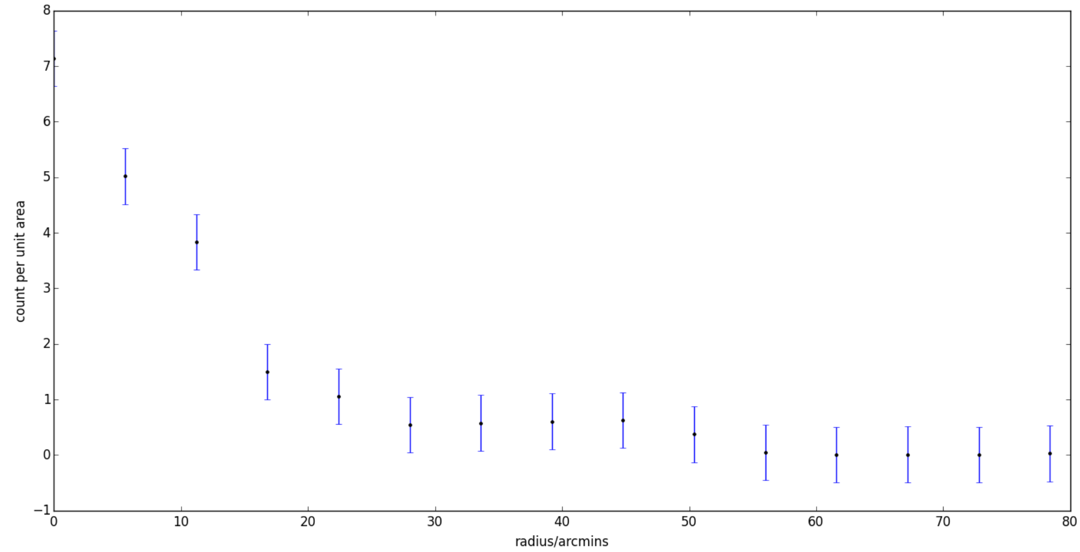

Figure 3: Density profile of the cluster plotted against position in arcmins

To find the density of the galaxy cluster the model of an isothermal gas sphere was used. For a gas sphere in hydrostatic equilibrium:

$$\frac{dp}{dr} = -\frac{GM_r}{r^2}\rho(r) \tag{6}$$

where

$$M_r = \int_0^r 4\pi r^2 \rho(r) dr \tag{7}$$

Also relating pressure and density,

$$p = \frac{kT}{m}\rho \tag{8}$$

where k is boltzmanns constant, T is the temperate of the gas, m is the mass of the particles in the gas and ρ is the density. Combining equations 6,7 and 8 the differential equation 9 is found.

$$\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{d\rho(r)}{dr}\right) + \frac{4\pi G m r^2}{kT}\rho(r) = 0 \tag{9}$$

To make the equation dimensionless some elements were defined as :

$$r = \alpha\xi \tag{10}$$

$$\alpha^2 = \frac{\langle\hat{V}^2\rangle}{12\pi G\rho_c} \tag{11}$$

$$\rho(r) = \rho_c \rho_1(\xi) \tag{12}$$

which leaves the equation in dimensionless form

$$\frac{d}{d\xi}\left(\frac{\xi^2}{\rho_1}\frac{d\rho_1(\xi)}{d\xi}\right) + \xi^2\rho_1 = 0 \tag{13}$$

This can then be solved numerically with the boundary conditions of

$$\rho_1(0) = 1 \tag{14}$$

$$\frac{d\rho_1(0)}{d\xi} = 0 \tag{15}$$

This then gives values as shown in Table 1 in the Tables section. These results were plotted to get the distribution shown in Figure 4.

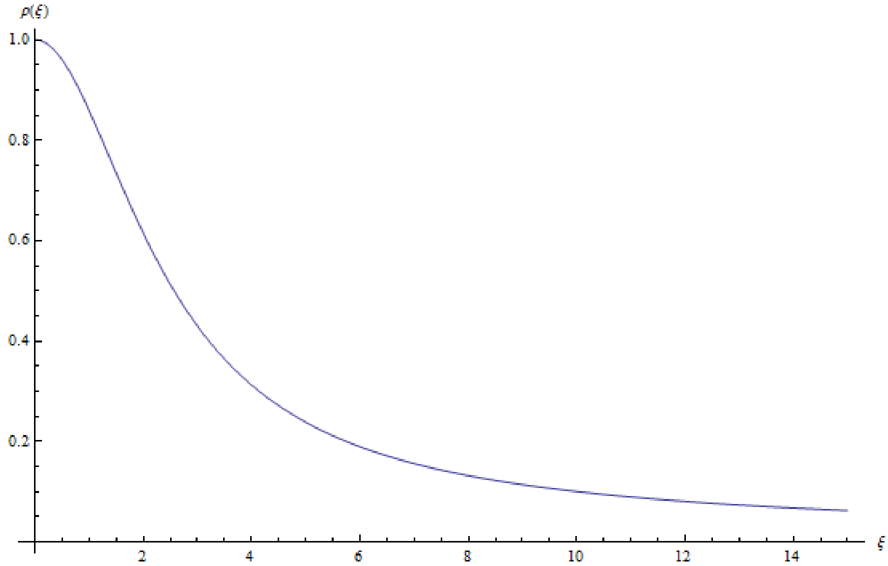

Figure 4: Plot of density profile of Isothermal gas sphere

This can already be seen to have a similar distribution to the density profile of the galaxy. However as the observed density profile of the galaxy found earlier was a projection onto a plane, it is not a true density of the cluster. Therefore the theoretical projected density, $\sigma_1(r_1)$, needs to be found from the theoretical density which came from the dimensionless equation, $\rho_1(\xi)$. This can be done by integrating the dimensionless equation numerically using Simpsons rule.

$$\int_a^b \rho(\xi)d\xi = \frac{h}{3}[\rho(\xi_1) + 4\rho(\xi_2) + 2\rho(\xi_3) + 4\rho(\xi_4) + ... + 4\rho(\xi_{n-1}) + \rho(\xi_n)] \tag{16}$$

where b and a run from the current radius, r_1 to the end of the data. This gives results shown in Table 3 in the tables section. The ξ also needs to be converted to radius r_1, this is done using r_1 as the projection on ξ onto the z plane.

$$r_1 = \sqrt{\xi^2 + z_1^2} \tag{17}$$

The calculated values of the projected density can be plotted on the same axis as the observed density profile. The solution of the isothermal gas sphere is infinite and the observed density has a finite value therefore the solution needs to be bound. This was done by subtraction around 0.3 from the theoretical solution. By then plotting both the theoretical and the observed density profile's it is possible to get a fit to the observed data, Figure 5.

The observed projected density $\sigma(r)$ differs from the theoretical, $\sigma_1(r)$, by a constant factor which was found using trial and error. Also due to the fact that $r = \alpha r_1$ this factor is also adjusted to fit the observed data. Now the fit has been applied as best as possible the values of

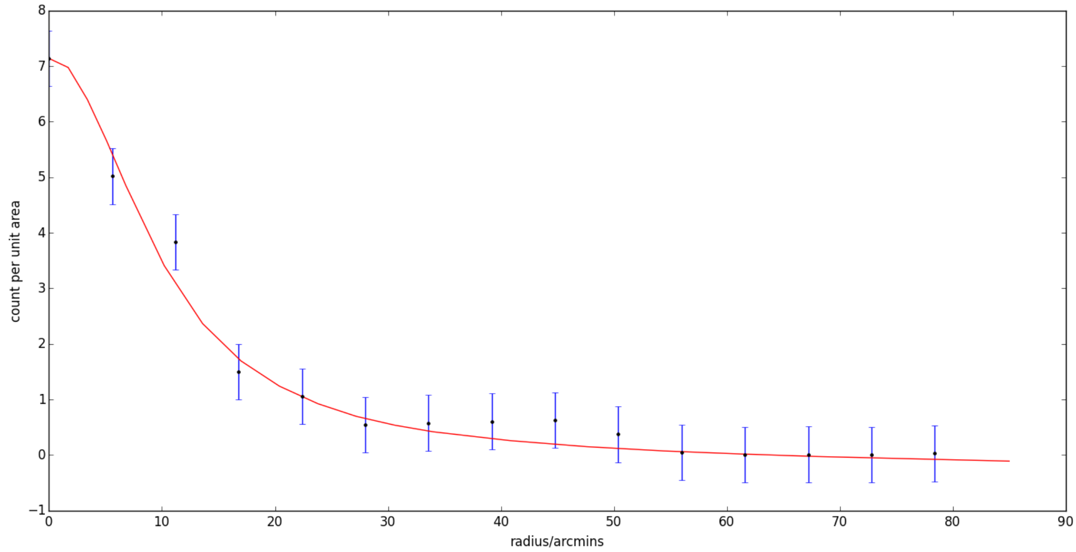

Figure 5: Density profile of the cluster plotted against position in arcmins

α and σ_c can be read off as:

$$\alpha = (3.4 \pm 0.1) arcmins$$

$$\sigma_c = (1.18 \pm 0.08)\ Galaxies/arcmin^2 = (3135 \pm 212)\ Gal/Mpc^2$$

by measuring where the profile of the theoretical density falls to half its central density, it is seen that it reaches this point at around 3α. This result can then be compared with the core radius of the Hydra cluster which is

$$3\alpha = (10.2 \pm 0.3) arcmins = (0.198 \pm 0.001)\ Mpc$$

$$R_I = (9.82 \pm 0.5)\ arcmins = (0.19 \pm 0.03)\ Mpc$$

The value of 3α can be seen to fall within the uncertainty of the measured radius of the core of the galaxy as expected.
By using the Virial Theorem it is possible to calculate the mass of the galaxy from the radius that has been worked out and the velocity dispersion of the galaxies. The gravitational potential energy is related to the kinetic energy by

$$\Omega = 2T \tag{18}$$

and by taking the kinetic energy of the cluster as:

$$T = \frac{3}{2} NM \langle \hat{V}^2 \rangle \tag{19}$$

where N is the number of galaxies, M is the average mass of a galaxy in the cluster and $\langle \hat{V}^2 \rangle$ is the velocity dispersion. The kinetic energy is 3 times as the true velocity dispersion is in three dimensions. The gravitational potential energy can be roughly worked out as:

$$\Omega = \frac{3}{5} \frac{GM^2}{R} \tag{20}$$

therefore by equation equations 18, 19 and 20 it is possible to find an expression for the mass of the cluster:

$$M_G = NM = 5\langle \hat{V}^2 \rangle \frac{R}{G} \tag{21}$$

The velocity dispersion can be worked out from given values of the velocities of the galaxies in the cluster. This is done by subtracting the mean value of the radial velocity from these given values, then by squaring the get the velocity dispersion, this is shown in Table 3 in the Table section. The velocity dispersion was calculated to

$$\langle \hat{V}^2 \rangle = 4.43 \cdot 10^{11} ms^{-1}$$

By then inputting the velocity dispersion and radius into equation 21 the mass of the cluster was found to:

$$M_H = (12.6 \pm 3) \cdot 10^{44} kg$$

Finally the estimate for the average universe density was calculated. This was done by taking the measured mass of the cluster and the fact that clusters are on average separated by $50Mpc$. By considering the mass of the galaxy as M_H and that there are 8 galaxies per $50Mpc^3$ the density can be found as.

$$\rho_{universe} = \frac{8 \cdot M_H}{V^3} = \frac{8 \cdot 12.6 \cdot 10^{44} kg}{(50Mpc)^3} = (8.064 \pm 1.9) \cdot 10^{40} kg/Mpc^3 \tag{22}$$

This value needs to be converted into units of $[kg/m^3]$ so that it can be compared with the critical density.

$$\rho_{universe} = (8.064 \pm 1.9) \cdot 10^{40} kg/Mpc^3 = (2.7 \pm 0.6) \cdot \ 10^{-27} kg/m^3 \tag{23}$$

The critical density of the universe is:

$$\rho_c = 7.9 \cdot 10^{-27} kg/m^3$$

therefore the calculated value from the clusters mass is within the order magnitude of the critical density. This shows how the isothermal sphere model is a good fit to the galaxy cluster.

3 Conclusion

By analysing an image taken of the Hydra cluster it is possible to determine its density profile and mass. Initially the background count of galaxies were looked at, from this count it is possible to determine the apparent magnitude of the faintest galaxy in the image and therefore, the lowest absolute magnitude of the galaxies in the count. These were found to be:

$$m = (16.08 \pm 0.5)$$

$$M = (-18.05 \pm 0.5)$$

To find the density profile of the galaxy the galaxies were counted in concentric rings and then divided by the area of the ring. The central density of the galaxy was found to be:

$$\sigma(0) = (6.81 \pm 0.5) \ Gal/arcmin^2 \tag{24}$$

However this data can be approximated by using the model of the isothermal gas sphere. By solving this model and finding the projected density profile onto a plane it is possible to fit this to the data. The model was modified by factors α and σ_c so that it would fit to the observed data better. These factors were found to be:

$$\alpha = (3.4 \pm 0.1) arcmins$$

$$\sigma_c = (1.18 \pm 0.08)\ Galaxies/arcmin^2 = (3135 \pm 212)\ Gal/Mpc^2$$

The inner radius of the cluster can be found at where the central density of the galaxy falls to half, this was found to:

$$R_I = (9.82 \pm 0.5)\ arcmins = (0.19 \pm 0.03)\ Mpc$$

also the radius of the cluster is found where the density falls to zero, this was found to be:

$$R_O = (63.7 \pm 0.5)\ arcmins = (1.24 \pm 0.03)\ Mpc$$

By using the Virial theorem an expression for the mass of the cluster could be found using the radius of the cluster and the average velocities of the galaxies within the cluster. The mass of the cluster was found to be:

$$M_H = (12.6 \pm 3) \cdot 10^{44} kg = (6.6 \pm 1.5) \cdot 10^{14} M_{\odot}$$

Assuming that all clusters in the universe are similar to that of the Hydra cluster. Then by knowing the mass of the cluster and that galaxies are on average separated by around $50 Mpc$ it is possible to find the density of the universe. This was calculated to:

$$\rho_{universe} = 7.9 \cdot 10^{-27} kg/m^3$$

This comes within the same order of magnitude as the critical density which implies that the isothermal sphere model is a good fit to the cluster of galaxies.

4 Tables

Table 1: EmdenLane solution

ξ	$\rho_1(r)$	ξ	$\rho_1(r)$	ξ	$\rho_1(r)$	ξ	$\rho_1(r)$	ξ	$\rho_1(r)$
0.0,	1.0,	2.5,	0.511991,	5.0,	0.238404,	7.5,	0.143406,	10.0,	0.100627,
0.125,	0.997403,	2.625,	0.489548,	5.125,	0.231215,	7.625,	0.140473,	10.125,	0.0991308,
0.25,	0.989701,	2.75,	0.468323,	5.25,	0.224388,	7.75,	0.137651,	10.25,	0.0976774,
0.375,	0.977151,	2.875,	0.448277,	5.375,	0.217898,	7.875,	0.134933,	10.375,	0.0962655,
0.5,	0.960158,	3.0,	0.429362,	5.5,	0.211724,	8.0,	0.132314,	10.5,	0.0948933,
0.625,	0.939243,	3.125,	0.411526,	5.625,	0.205848,	8.125,	0.129789,	10.625,	0.0935592,
0.75,	0.915005,	3.25,	0.394714,	5.75,	0.20025,	8.25,	0.127354,	10.75,	0.0922618,
0.875,	0.888081,	3.375,	0.378872,	5.875,	0.194913,	8.375,	0.125004,	10.875,	0.0909996,
1.0,	0.859112,	3.5,	0.363943,	6.0,	0.189822,	8.5,	0.122735,	11.0,	0.0897714,
1.125,	0.828709,	3.625,	0.349873,	6.125,	0.184961,	8.625,	0.120543,	11.125,	0.0885757,
1.25,	0.797435,	3.75,	0.336609,	6.25,	0.180318,	8.75,	0.118425,	11.25,	0.0874114,
1.375,	0.765787,	3.875,	0.3241,	6.375,	0.175879,	8.875,	0.116377,	11.375,	0.0862773,
1.5,	0.734188,	4.0,	0.312299,	6.5,	0.171632,	9.0,	0.114395,	11.5,	0.0851723,
1.625,	0.702989,	4.125,	0.301159,	6.625,	0.167567,	9.125,	0.112478,	11.625,	0.0840953,
1.75,	0.672468,	4.25,	0.290638,	6.75,	0.163672,	9.25,	0.110622,	11.75,	0.0830453,
1.875,	0.64284,	4.375,	0.280694,	6.875,	0.15994,	9.375,	0.108825,	11.875,	0.0820214,
2.0,	0.614259,	4.5,	0.271291,	7.0,	0.15636,	9.5,	0.107083,	12.0,	0.0810227,
2.125,	0.586834,	4.625,	0.262392,	7.125,	0.152924,	9.625,	0.105394,	12.125,	0.0800482,
2.25,	0.56063,	4.75,	0.253965,	7.25,	0.149624,	9.75,	0.103757,	12.25,	0.0790971,
2.375,	0.53568,	4.875	0.245978,	7.375,	0.146454,	9.875	0.102169,	12.375,	0.0781686,

Table 2: Table of projection $\sigma_1(r_1)$

r_1	$\sigma_1(r_1)$	r_1	$\sigma_1(r_1)$	r_1	$\sigma_1(r_1)$	r_1	$\sigma_1(r_1)$
0	6.0557	8	0.86	50	0.139	200	0.0216
0.5	5.924	9	0.732	60	0.0843	250	0.0185
1	5.458	10	0.636	70	0.0749	300	0.0161
1.5	4.86	12	0.508	80	0.0657	350	0.0143
2	4.212	14	0.42	90	0.0614	400	0.0128
3	3.048	16	0.358	100	0.052	450	0.0106
4	2.206	18	0.314	120	0.0451	500	0.0091
5	1.664	20	0.277	140	0.0397	600	0.0079
6	1.298	30	0.21	160	0.0321	700	0.007
7	1.044	40	0.19	180	0.0259	800	0.0063

Table 3: Table of velocity distributions in cluster

NGC	V(km/s)	$V-\bar{V}(km/s)$	$(V-\bar{V})^2(km/s)^2$
3309	3801	301	90601
3308	3687	187	34969
3311	3575	75	5625
3336	3689	189	35721
IC2597	2738	-762	580644
3312	2512	-988	976144
3285	3049	-451	203401
3285A	3049	-451	203401
3285B	2868	-632	399424
3305	4549	1049	1100401
3314	2635	-865	748225
3315	4555	1055	1113025
3316	3752	252	63504
3307	3616	116	13456
IC629	2461	-1039	1079521

References

[ZW50] Zwicky, F., Physical Characteristics of the Cancer Cluster of Galaxies, 1950

[CH58] S. Chandrasekhar, An Introduction to the Study of Stellar Structure, 1958