PH3010 - Solar Limb Darkening

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Abstract

By observing the limb darkening of the sun, it was possible to measure the temperature as a function of optical depth ,τ_λ, for different wavelengths, λ. With no filter the surface temperature of the sun rose from $(5710 \pm 120)K$ at the surface to $(6653 \pm 120)K$ at an optical depth of 1. The red, green and blue wavelengths ,$\sim 699nm$, $\sim 501nm$ and $\sim 426nm$ respectively, followed a similar distribution as a function of optical depth displaying how the temperature of the photosphere increases with optical depth.

1 Introduction

It can be seen from observing the sun that towards the limbs it appears to get darker. By measuring this intensity change it is possible to find out the temperature distribution of the sun. This limb darkening can be explained by looking at where in the photosphere the photons have come from. Photons that escape from a smaller radius of the photosphere will have originated in a hotter region; therefore will have a higher intensity. Photons that originate from a larger radius come from a cooler part of the photosphere; therefore a lower intensity. There is a given probability that a photon will escape from the sun which depends on its optical depth into the sun. Figure 1 shows how photons that have the same probability of escaping nearer the surface come from a different part of the photosphere to ones nearer the center. At the center of the sun the photons originate from a smaller radius and a hotter part of the photosphere. The photons near the edge come from a larger radius and a cooler part of the photosphere.

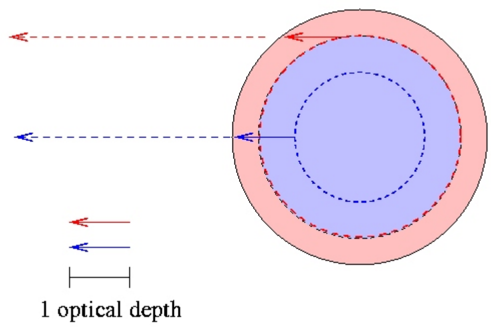

Figure 1: Optical depth at center and at limb [3]

This explains why the suns limbs are darker as the photos escaping near the edge are 'cooler'. This gives an intensity distribution across the sun. This intensity distribution of the sun depends

on both the absorption and emission probability of the photon.
The intensity dependence on emission is given by:

$$dI_\lambda = j_\lambda \rho ds \tag{1}$$

where dI_λ is the change in intensity, j_λ is the absorption coefficient ρ is the density of the matter and ds is the thickness element of matter. The intensity dependence on absorption is:

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds \tag{2}$$

where κ_λ is the absorption coefficient, I_λ is the intensity and the other symbols are the same as equation (1). At this point the source function can also be defined as the ratio of the emission and absorption coefficients:

$$S_\lambda = \frac{j_\lambda}{\kappa_\lambda} \tag{3}$$

where S_λ is the source function. By combining equations (1), (2) and (3) and rearranging the total intensity distribution can be found.

$$\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = S_\lambda - I_\lambda \tag{4}$$

This is currently a function of the thickness of matter ds however it needs to be changed into a function of optical depth. To do this the optical depth needs to be defined.

$$d\tau_\lambda = -\kappa_\lambda \rho ds \tag{5}$$

where $d\tau_\lambda$ is the change in optical depth. The equation of the intensity can then be changed into a function of optical depth (τ_λ) and position (μ):

$$\mu \frac{dI_\lambda(\tau_\lambda, \mu)}{d\tau_\lambda} = I_\lambda(\tau_\lambda, \mu) - S_\lambda(\tau_\lambda) \tag{6}$$

where τ is the optical depth.
The position on the suns disc μ is defined as:

$$\mu = \sqrt{1 - \left(\frac{r}{R}\right)^2} \tag{7}$$

where R is the radius of the sun and r is the position we look at on the suns disc. By solving equation 6, the source function of the star can be found from the intensity distribution. As the source function cannot be solved explicitly, it can be approximated by a series expansion of:

$$S_\lambda(\tau_\lambda) = I_\lambda(0,1) \sum a_{\lambda n} \tau_\lambda^n \tag{8}$$

where $I_\lambda(0,1)$ is the measured intensity at the center of the center of the sun, $a_{\lambda n}$ is the coefficient of the optical depth τ_λ. A good approximation of this expansion is expanding it to n=2.
The method of least squares can be used to apply a fit to the intensity distribution of the sun and by picking a function similar to that of the source function the values of $a_{\lambda n}$ can be found. This function looks like:

$$f(\mu, a) = a_0 + a_1 \mu + 2a_2 \mu^2 \tag{9}$$

The values of $a_{\lambda n}$ found from the fit can then be used to find the source function by putting the values into the expansion in equatiom 8.

Assuming the photosphere is in thermal equilibrium the intensity of the sun can be given by the Planck black body distribution:

$$I_\lambda = B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1} \tag{10}$$

where $B_\lambda(T)$ is the Planck function, h is Plancks constant, c is the speed of light, λ is the wavelength of light, k is the Boltzmann constant and T is temperature. Due to the assumption that the photosphere is in local thermal equilibrium, the temperature can be defined for a particular depth such that the source function is equal to the black body distribution. By rearranging the equation the temperature as a function of optical depth can be found.

$$T(\tau_\lambda) = \frac{hc/k\lambda}{\ln(1+\frac{2hc^2}{\lambda^5 S_\lambda(\tau_\lambda)})} \tag{11}$$

where $T(\tau_\lambda)$ is the temperature as a function of optical depth.

2 Experiment

2.1 Data collection

To collect the data a Meade LX200 10" telescope was used with a solar filter. To capture the images a Modified Logitech Quickcam Pro 3000 camera was used alongside a red, blue, green and clear filter. The sun was initially centred in the telescope with the clear/no filter. This meant that the exposure could be adjusted to make sure that the images were not saturated. It was found that at a exposure of 1/450 seconds the image did not saturate. Although due to the changing cloud cover this did fluctuate. The time taken to slew across the diameter of the sun was recorded as 15s, this same slew time was taken either side of the sun for a background reading. The telescope was slewed 15s past the edge of the sun, the telescopes tracking was then turned of and the sun was allowed to drift across the field of view with images being taken at 1s intervals. This process was repeated for all the different filters. Due to poor seeing conditions the data that was taken was was not usable as the cloud cover interfered with our data. As can be seen of the intensity distributions in Figure 2, this data is clearly not good enough to analyse and use to find source functions and temperature.

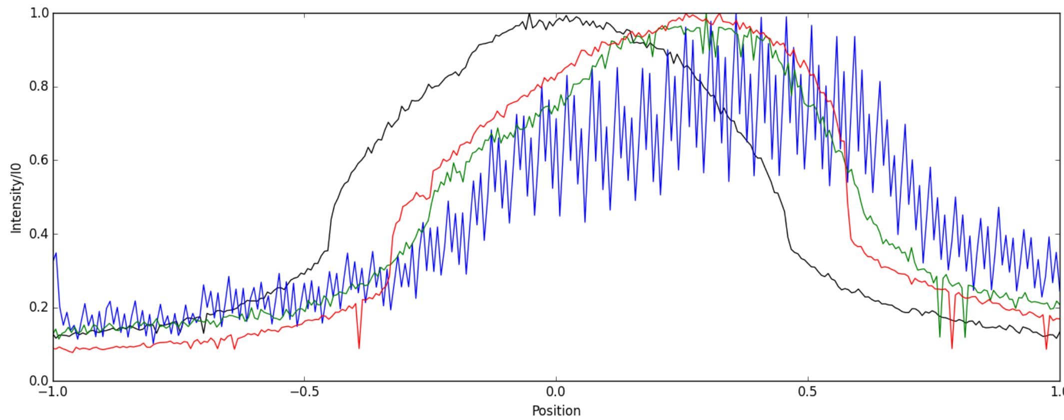

Figure 2: Plots of bad data due to cloud cover

2.2 Data Analysis

Once a using a useful set had been acquired the analysis could then be carried out. The images were imported into the computer and a filelist was created so that the images could be easily loaded into python. An area of around 3x3 pixels at the center of the image was analysed; the area was positioned so that a sunspot did not pass through it. The pixel values of the pixels in this area were summed so that this intensity distribution could be plotted. Figure 3 shows the intensity distribution of the sun and the background readings.

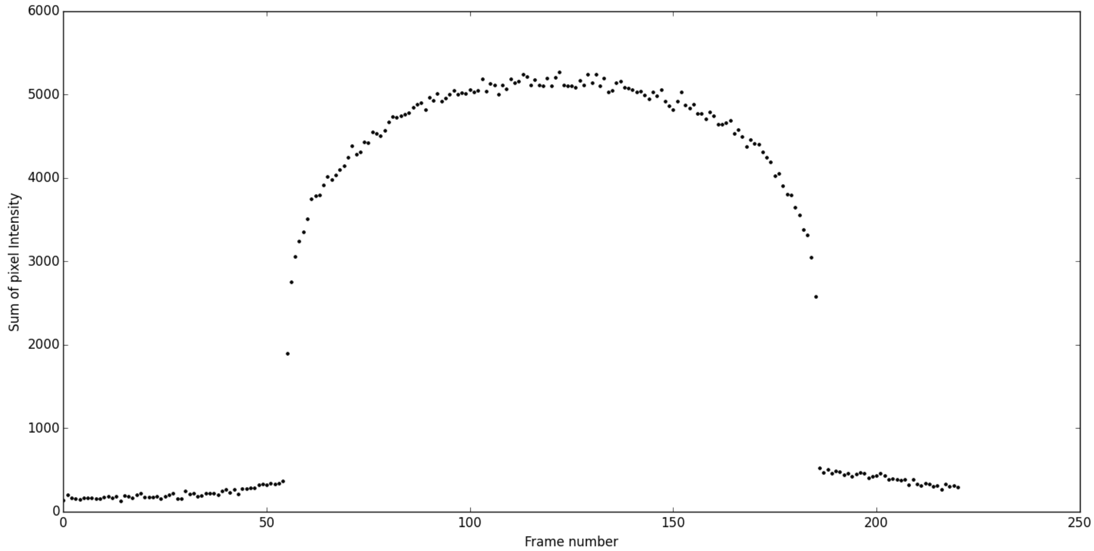

Figure 3: Plot of intensity/I0 against the frame number for the clear filter

The pixel values however are not particularly useful in finding out any physical quantities so they were normalised so that at the center of the star the values are 1; this was done by dividing by the pixel value at the center of the sun. The aim was to get a fit of the intensity across the star however the background readings needed to be removed before this was possible. To do this the intensity distribution was differentiated so that the edges of the star could be found. By plotting the differential in Figure 4 the edges of the sun are clear, they are located at the maximum and minimum peaks as this was where the maximum gradient is. The data could then be clipped at these points.

The frame number is also not particularly useful so the x-axis was scaled from -1 to 1 at the edges of the star; this means that it is a plot of I/I0 against position on the suns disc. The next task was to fit the intensity plot of the sun, for this the curvefit package was used in python alongside the fit function that was mentioned earlier in eq 8. This fit the intensity as shown in Figure 5:

This plot also shows errors for the intensities, these were found by taking a set of images at the same point at the center of the sun. Then a 3x3 pixel square was taken similar to before and the intensities measured, they were scaled by using the same factor as before. The intensities were put into the equation:

$$\sigma = \sqrt{\frac{n}{n-1}(\overline{I^2} - \overline{I}^2)} \tag{12}$$

where σ is the standard deviation, n is the number of images taken, $\overline{I^2}$ is the average of the squared intensities and $\overline{I}^2$ is the average of the intensities squared.

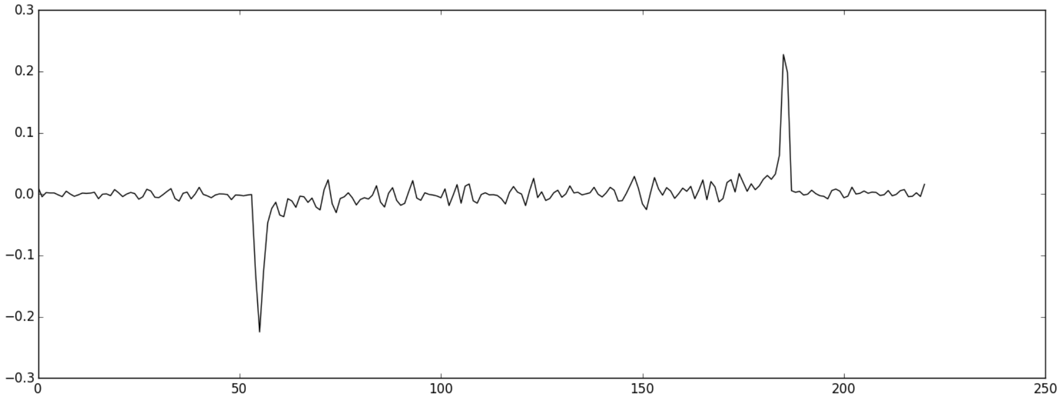

Figure 4: Differentiated plot of the intensity of the clear filter

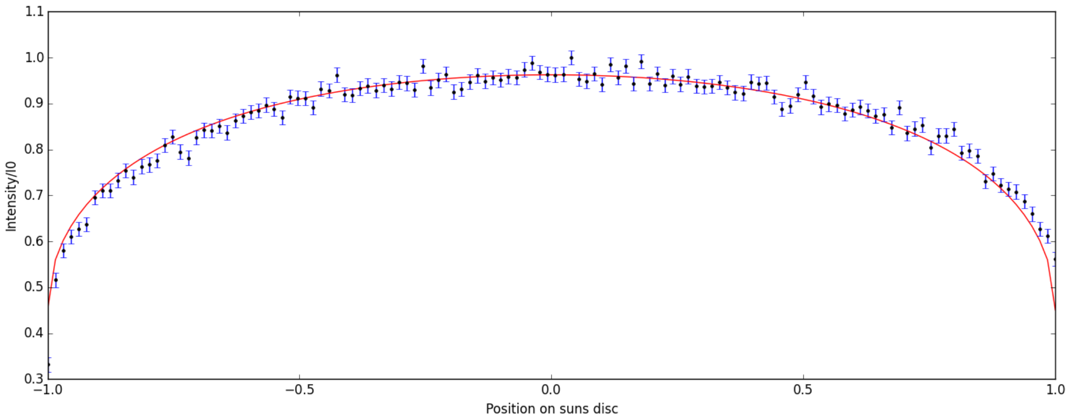

Figure 5: Fit of least squares fitted to Intensity plot for the clear filter

The fit to the intensity plot gave the values of a_{cn} as:

$$a_{c0} = 0.46787619$$

$$a_{c1} = 0.67104811$$

$$a_{c2} = -0.06948355$$

Here λ has been replaced with c which means the clear filter, it will be r,g and b for the red, green and blue filters respectively. There are also errors on these values of a_{c0}, these were found from the curvefit program. These were found to be:

$$a_{c0} = 0.45025742 \pm 0.01286953$$

$$a_{c1} = 0.64577851 \pm 0.04244098$$

$$a_{c1} = -0.06686701 \pm 0.01643566$$

These values can then be used to find the source function by plugging them into the source function expansion in eq 7. Photons that come from the edge of the star come from an optical depth of 0 and at the center of the star an optical depth of 1.

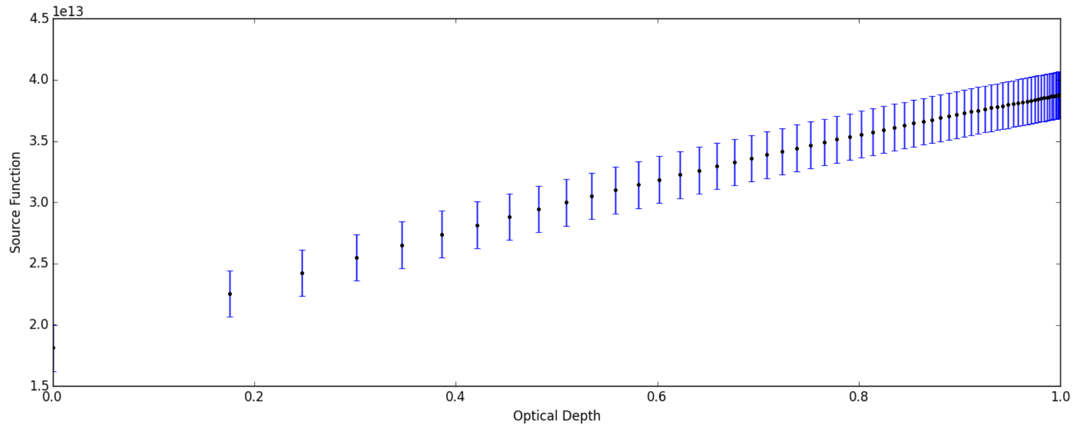

Figure 6: Source function plotted against optical depth for the clear filter

This means that the values of μ correspond the the optical depth τ therefore the source function is plotted against optical depth and not position μ.
The errors for the source function were found by propagating the errors from the values of a_{c0}. To calculate the error the propagation formula was used which is [4]:

$$\sigma_f = \sqrt{\left(\frac{df}{dx}\right)^2 \cdot \sigma_x^2 + \left(\frac{df}{dy}\right)^2 \cdot \sigma_y^2 + \left(\frac{df}{dz}\right)^2 \cdot \sigma_z^2} \tag{13}$$

where σ_f is the error in function f, σ_x is the uncertainty in measurement x etc.
By inputting the source function equation into this the error propagation becomes:

$$\sigma_S = \sqrt{\sigma_{a_0}^2 + \sigma_{a_1}^2 + \sigma_{a_2}^2} \tag{14}$$

where σ_S is the uncertainty in the source function, and σ_{an} is the uncertainty in each of the parameters.
By inputting all of the data the errors came out as shown in Figure 6. To get the temperature against optical depth the source function is put into the temperature equation, eq 11.
This is then plotted against optical depth as Figure 7 shows.

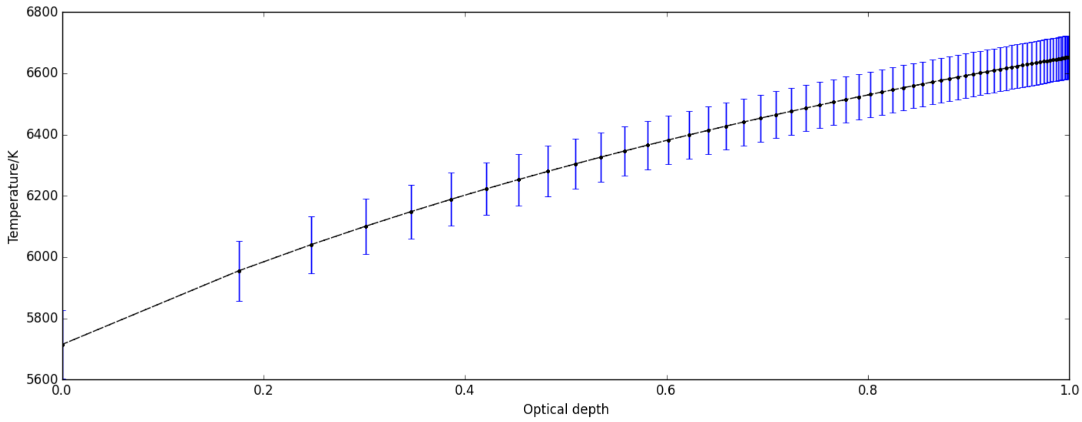

Figure 7: Temperature plotted against optical depth for the clear filter

The errors for the temperaure were propagated using the equation 13 the same as before. The differential of the temperature equation is:

$$\frac{dT}{dS} = \frac{2c^3h^2}{\left(kl^6s^2\left(\frac{2c^2h}{l^5s}+1\right)\log\left(\frac{2c^2h}{l^5s}+1\right)^2\right)} \tag{15}$$

This can be in-putted into equation 13 to give:

$$\sigma_T = \sqrt{\left(\frac{dT}{dS}\right)^2\sigma_S^2} \tag{16}$$

These values of the errors are plotted in Figure 7. The temperature plot, Figure 7, shows how the temperature rises from $\sim 5700K$ at an optical depth of 0 to $\sim 6700K$ at an optical depth of 1. The surface temperature is measured to:

$$T_{surface} = (5710 \pm 120)K$$

$$T_{max} = (6653 \pm 120)K$$

where T_{max} is the temperatures at an optical depth of 1.
The surface temperature of the sun has been calculated to $\sim 5800K$ therefore this value comes well within the uncertainty of the measured value of the surface above. This process was repeated for the red, green and blue filters , at wavelengths $\sim 699nm$, $\sim 501nm$ and $\sim 426nm$ respectively, to give temperature plots as shown in Figure 8.

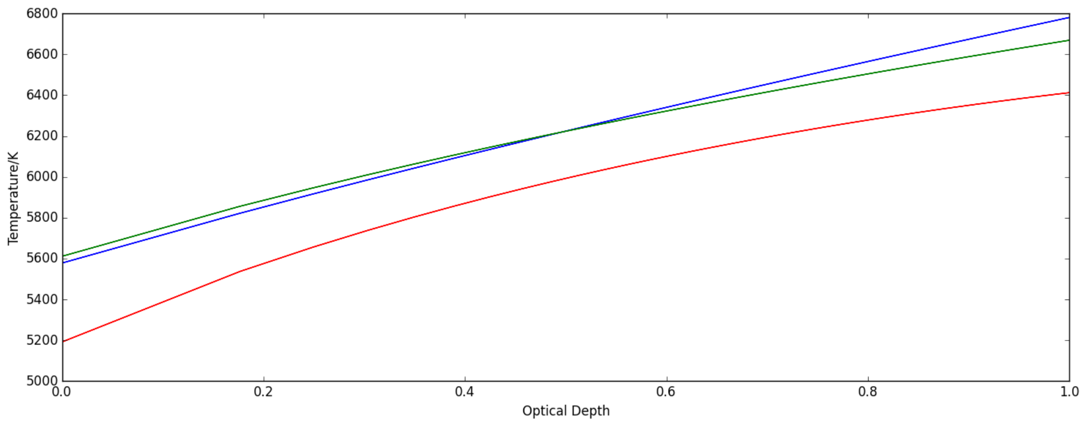

Figure 8: Temperature plotted against optical depth for all filters

For the red filter the temperatures went from:

$$T_{r-min} = 5190 \pm 160$$

$$T_{r-max} = 6412 \pm 160$$

For the green filter:

$$T_{g-min} = 5578 \pm 130$$

$$T_{g-max} = 6780 \pm 130$$

For the blue filter:

$$T_{b-min} = 5611 \pm 116$$

$$T_{b-max} = 6669 \pm 116$$

It can be seen in Figure 8 that the red and green filters follow a similar curve whereas the blue filter does not, this may be due to the fact that blue light is scattered more in the earth's atmosphere therefore the image may be distorted. This may be overcome by taking a background image of the sky and removing from the blue filters image. The data recorded shows how the temperature of the photosphere increases with optical depth. This is the same for all the filters although the temperature is lower for the red filter as it has a longer wavelength. The limb darkening is also greatest for the blue wavelength which is why the limbs appear to be redder that the center.

3 Conclusion

By taking images across the suns surface it was possible to extract the intensity distribution of the sun which shows its limb darkening. By applying a fit to this data it was possible to find parameters that could be inputted into an equation for the source function, these were $a_{\lambda 0} = 0.46787619$,$a_{\lambda 1} = 0.67104811$, $a_{\lambda 2} = -0.06948355$. Then by using Plancks law of black body radiation this source function was used to plot the temperature as a function of optical depth of the sun. This gave a plot which went from around:

$$T_{surface} = (5710 \pm 120)K$$

$$T_{max} = (6653 \pm 120)K$$

As the surface temperature of the sun is measured to $\sim 5800K$ this comes within the uncertainty of the measurement above. The data shows how the temperature of the photosphere increases with the optical depth this would make sense as the temperature of the sun is higher nearer the center.

References

[1] Erika Bohm-Vitense *Introduction to Stellar astrophysics* 1989.

[2] Dina Prialink *Introduction to the theory of Stellar structure and evolution* 2000.

[3] *http://spiff.rit.edu/classes/phys440/lectures/limb/limb.html* optical depth image.

[4] John R Taylor *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements* 1996.