1 Offset Parameters

The alignment of any particular image to a given specific reference frame (such as the first image in a sequence) can be defined by a shift in origin coordinates, Δx and Δy, combined with a clockwise rotation about the origin, θ. An example of this offset is shown in Fig. 1.

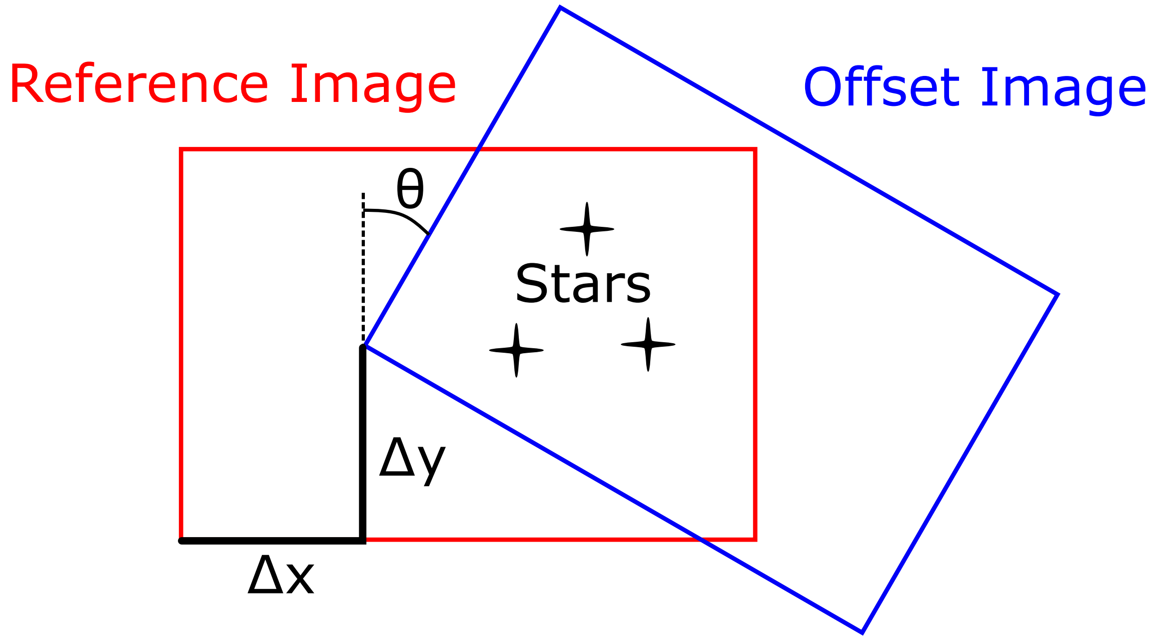

Figure 1: An example of how to define the offset of an image in relation to a reference image using the parameters Δx, Δy and θ, in order to match stars common to both images.

Given these offset parameters, any position in the offset image can be mapped onto an equivalent position in the reference frame using the following matrix transformation:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \tag{1}$$

where x and y are the original coordinates in the offset image, and x' and y' are the equivalent coordinates in the reference image.

The parameters of an alignment are possible to calculate, given that area of overlap between the images is large enough (area $\geq 95\%$) such that most stars visible in one image are also visible in the other. This can be done using the positions of a selection of bright stars in both images, which can be found by running the star finding algorithm with a high threshold (such as the mean of the image multiplied by 1.5). Accurate determination of background is not required for this.

Assuming both images are taken under similar conditions and with identical filters, matching stars can be done in relation to relative flux values; stars can be matched simply by ordering from brightest to dimmest, since the same stars with the same relative fluxes appear in both images. However, matching breaks down in the case that at least one bright star near the edge of one image do not appear in the other image.

Given that at least the first and second brightest stars are matched between images, an approximation can be made for rotation θ using the equation:

$$\theta = \tan^{-1}\left(\frac{x'_1 - x'_2}{y'_1 - y'_2}\right) - \tan^{-1}\left(\frac{x_1 - x_2}{y_1 - y_2}\right) \tag{2}$$

where the subscripts 1 and 2 denote the first or second brightest stars in either image. Given this approximation, it is then possible to approximate Δx and Δy using a rearrangement of the coordinate transformations from above:

$$\Delta x = x' - x\cos\theta - y\sin\theta \tag{3}$$

$$\Delta y = y' + x\sin\theta - y\cos\theta \tag{4}$$

These approximations can be used to refine the matching between stars; the transformation given in Eqn. 1 using the approximated parameters can be used to determine whether a star in one image would be within the bounds. Better matching is necessary for an improvement on the method of calculating offset parameters.

The approximations only take the matching of two stars into account at most, and hence may not be accurate for all stars in the image. Determining the optimal offset parameters can be done using a variation of the method of least squares to fit parameters. This requires defining a χ^2 statistic as:

$$\chi^2\left(\vec{\theta}\right) = \sum_{i=1}^{N}\left[\frac{\left(x'_{m,i} - x'_{\lambda,i}\left(x_i, y_i; \vec{\theta}\right)\right)^2}{\sigma_{x,i}^2} + \frac{\left(y'_{m,i} - y'_{\lambda,i}\left(x_i, y_i; \vec{\theta}\right)\right)^2}{\sigma_{y,i}^2}\right] \tag{5}$$

where $x'_{m,i}$ and $y'_{m,i}$ are the measured star coordinates in the reference image; $x'_{\lambda,i}$ and $y'_{\lambda,i}$ are the coordinates of a matched star in the the offset image which have had the transformation applied; $\sigma_{x,i}$ and $\sigma_{y,i}$ are the associated errors for coordinates in the reference images (acting as weights for the method); and $\vec{\theta} = (\Delta x, \Delta y, \theta)$.

Optimal values for the transformation parameters are found by minimising the value of the χ^2 in Eqn. 5. Errors for the parameters found are estimated using the inverse Hessian matrix of the χ^2 function. This method was applied to the blue filter Dragonfly Cluster images taken in observation 2, the results of which are shown in Table 1.

Table 1: Offset parameters and χ^2 statistics for the 15 blue filter images of the Dragonfly Cluster taken in observation 2, determined in relation to the first image in the set. The number of degrees of freedom is equal to the number of star positions compared minus three.

Image	Δx (pixels)	Δy (pixels)	θ (degrees)	χ^2_{min}	D.O.F.	Reduced χ^2_{min}
2	7.1390 ± 0.0117	-1.8594 ± 0.0112	-0.00906 ± 0.00049	9943.7	28	355.13
3	11.397 ± 0.012	-3.7744 ± 0.0112	-0.02657 ± 0.00049	8200.6	28	292.88
4	14.0218 ± 0.012	-4.7089 ± 0.0113	-0.04411 ± 0.00049	68.208	25	2.7283
5	17.506 ± 0.012	-5.8344 ± 0.0112	-0.04923 ± 0.00049	20880	28	745.72
6	21.620 ± 0.012	-7.5881 ± 0.0112	-0.05764 ± 0.00049	16735	28	597.69
7	24.256 ± 0.012	-7.7033 ± 0.0112	-0.06219 ± 0.00049	16620	28	593.56
8	28.106 ± 0.012	-8.6047 ± 0.0113	-0.06801 ± 0.00050	44829	28	1601.0
9	31.897 ± 0.012	-10.683 ± 0.011	-0.07739 ± 0.00050	41588	25	1663.5
10	33.821 ± 0.012	-11.133 ± 0.011	-0.08538 ± 0.00050	63259	28	2259.2
11	37.260 ± 0.012	-12.575 ± 0.011	-0.09326 ± 0.00050	74726	28	2668.8
12	40.398 ± 0.012	-12.162 ± 0.011	-0.10820 ± 0.00050	74264	28	2652.3
13	43.348 ± 0.012	-14.825 ± 0.011	-0.11788 ± 0.00050	74731	28	2669.0
14	45.774 ± 0.012	-14.696 ± 0.011	-0.13220 ± 0.00050	74730	28	2668.9
15	50.853 ± 0.012	-18.233 ± 0.011	-0.18131 ± 0.00050	78525	28	2804.5