CALCULATIONS METHODS FOR HADRON COLLIDERS: NEXT-TO-LEADING ORDER CROSS SECTION FOR $q\bar{q} \longrightarrow e^-e^+$

Sayyed Farbod Rassouli

Supervisor: **Dr. Nikolas Kauer**, Department of Physics

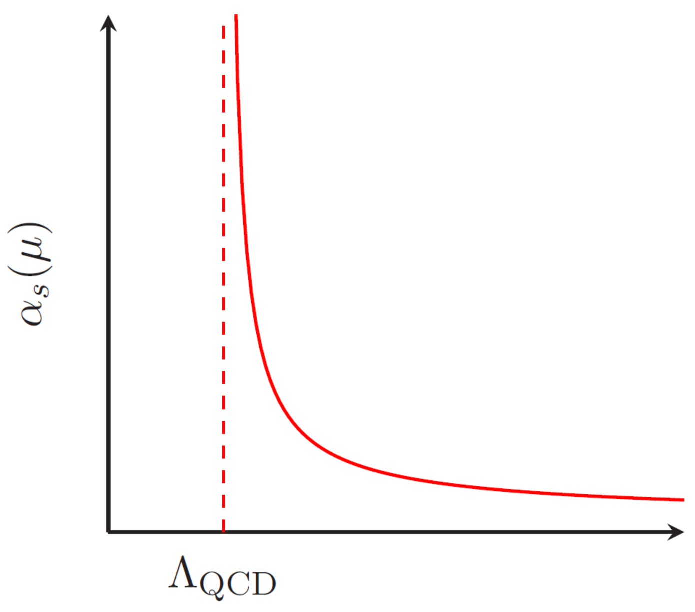
ROYAL HOLLOWAY UNIVERSITY OF LONDON

Abstract

Different mathematical methods are used and shown in this project for the calculation of the partonic-level process $q\bar{q} \longrightarrow e^-e^+$ at leading order (LO) and next-to-leading order (NLO) in perturbative Quantum Chromodynamics (QCD). Concluding that despite the non-perturbative nature of the proton it is possible to make important predictions for the hadron colliders.

Introduction and general formula for LO and NLO

An important part for understanding why perturbative QCD exists, resides in the feature of asymptotic freedom. This is shown in the figure below:



Where it is possible to understand at high scale (Λ) the coupling (α_s) become smaller. This contrary to QED makes possible at high scales to apply perturbation theory in the following way:

$$\sigma(Q^2) \sim \alpha_s^n \left[\sigma^{\mathrm{LO}} + \alpha_s \sigma^{\mathrm{NLO}} + \alpha_s^2 \sigma^{\mathrm{NNLO}} + \cdots\right]. \qquad (1)$$

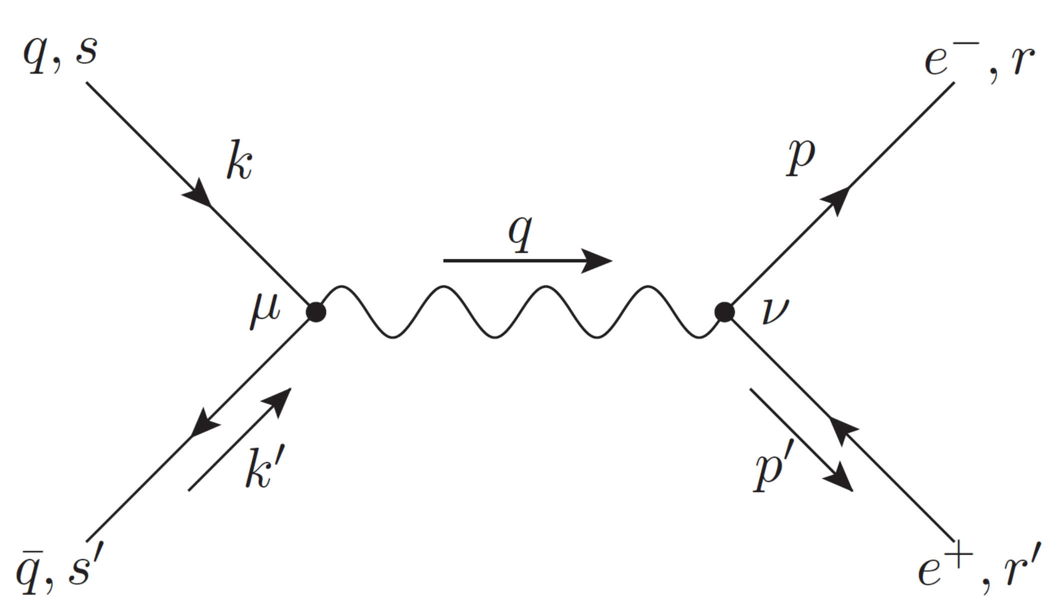
Therefore the form of the cross sections for LO and NLO are the following:

$$\sigma_{q\bar{q}}^{\mathrm{LO}} = \int d\Pi_2 \sum |\mathcal{M}_B|^2 \qquad (2)$$

$$\sigma_{q\bar{q}}^{\mathrm{NLO}} = \int d\Pi_3 \sum |\mathcal{M}_R|^2 + \int d\Pi_2 \sum |\mathcal{M}_V|^2 \qquad (3)$$

Where Π_n is the phase space, R and V are the real and virtual term.

Leading Order Calculation



$$i\mathcal{M}_B(q\bar{q} \longrightarrow e^-e^+) = \bar{v}^{s'}(k')(-iQ_f(-e)\gamma^\mu)u^s(k)(\frac{-ig_{\mu\nu}}{q^2})\bar{u}^r(p)(-ie\gamma^\nu)v^{r'}(p')$$

This represents the amplitude of the Feynman diagram represented in this section. The two-body phase space has been calculated:

$$\int d\Pi_2 = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2p^0}\frac{d^3p'}{(2\pi)^3}\frac{1}{2p'^0}(2\pi)^4\delta^{(4)}(k + k' - (p + p')) \qquad (4)$$

$$= \int d\Omega \frac{|\vec{p}\,|}{16\pi^2 E_{\mathrm{cm}}} \qquad (5)$$

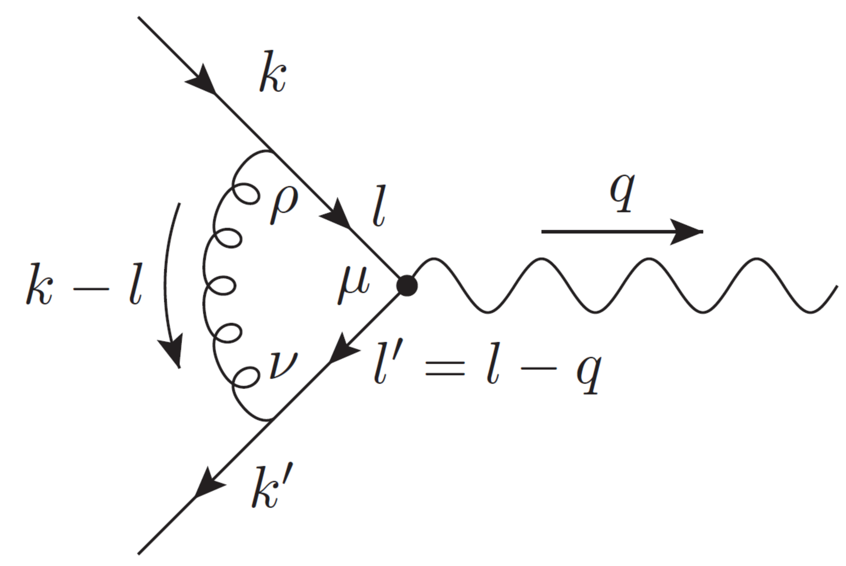
Has been evaluated the amplitude modulus square:

$$\frac{1}{12}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{8Q_f^2 e^4}{3q^4}\left[(k\cdot p)(k'\cdot p') + (k\cdot p')(k'\cdot p)\right] \qquad (6)$$

Now combining the phase space and the amplitude squared in the kinematics settings of the hadron collider the LO cross section is

$$\sigma_{q\bar{q}}^{\mathrm{LO}} = \frac{Q_f^2 e^4}{36\pi E_{cm}^2} \qquad (7)$$

Virtual correction term



$$i\mathcal{M}_V(q\bar{q} \longrightarrow e^-e^+) = \frac{ieQ_f}{q^2}(\bar{v}^{s'}(k')(\gamma^\mu + \delta\Gamma^\mu(k', k))u^s(k))(\bar{u}^r(p)\gamma_\mu v^{r'}(p'))$$

Where $\delta\Gamma^\mu$ represents the loop correction defined as:

$$\int \frac{d^4l}{(2\pi)^4}\frac{-ig_{\nu\rho}}{(k-l)^2 - \mu^2 + i\epsilon}(ig\gamma^\nu)\frac{il'}{l'^2 + i\epsilon}(-iQ_f(-e)\gamma^\mu)\frac{il}{l^2 + i\epsilon}(ig\gamma^\rho) \qquad (8)$$

By using a new variable $b = l - (yq + zk)$ is possible to simplify Eq.8 but for performing the integral has been used the **Wick rotation** method. This method consist changing the calculation from Minkowski to Euclidean space:

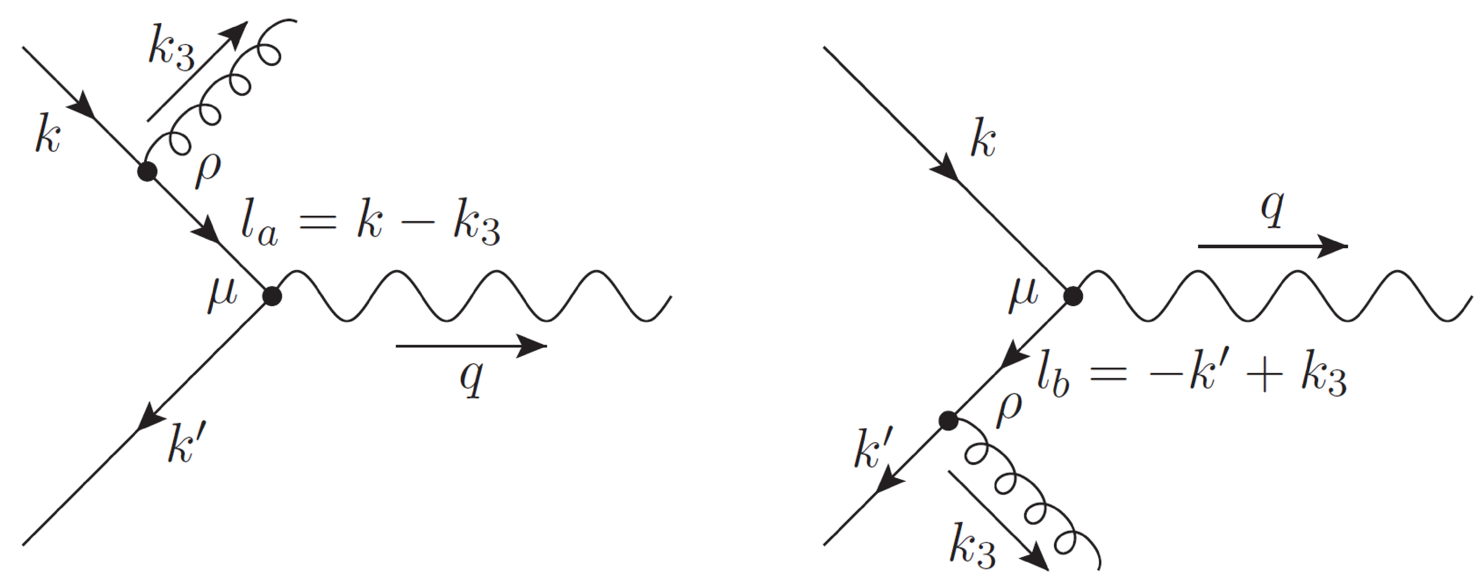
$$\int \frac{d^4b}{(2\pi)^4}\frac{1}{(b^2 - \Delta + i\epsilon)^3} = \frac{-i}{(2\pi)^4}\int d\Omega_4 \int_0^\infty d|\tilde{b}| \frac{|\tilde{b}|^3}{(|\tilde{b}|^2 + \Delta)^3} \qquad (9)$$

Where $|\tilde{b}|$ is the new 4 momentum in Euclidean space. By performing the **Pauli-Villars** regulation and applying $\delta F_1(q^2) \longrightarrow \delta F_1(q^2) - \delta F_1(0)$ it is possible to find the virtual cross section:

$$\sigma^V(q\bar{q} \longrightarrow e^-e^+) = \frac{e^4}{36\pi\hat{s}^2}\cdot\Bigg|Q_f + \frac{2g^2Q_f}{(4\pi)^2}\int_0^1 dy \int_0^{1-y} dz \times$$

$$\times \left[\log\left(\frac{z\mu^2}{-y(1-y-z)s + z\mu^2}\right) + \frac{(y+z)(1-y)s}{-y(1-y-z)s + z\mu^2}\right]\Bigg|^2 \qquad (10)$$

Real correction term



$$i\mathcal{M}_R = Q_f(-ie)^2(-ig)\epsilon_\rho(k_3)\bar{v}(k')\left[\gamma^\mu\frac{i}{k - k_3}\gamma^\rho - \gamma^\rho\frac{i}{k' - k_3}\gamma^\mu\right]u(k)$$

The three-body phase space has been calculated:

$$\int d\Pi_3 = \frac{q^2}{128\pi^3}\int dx_1 dx_2 \qquad (11)$$

The calculation for the cross section follows the mathematical tool used for LO calculation.

$$l_a^2 \approx |\vec{k}||\vec{k}_3|(1 - \cos\theta), \qquad (12)$$

Two type of divergences arises: $|\vec{k}_3|$ is zero (Soft divergence, gluon mass is 0) or where $\cos\theta = 1$ (collinear divergence, perpendicular or parallel to the quark). These divergence will be cancelled by the virtual term.

Conclusion

In conclusion has been possible to illustrate important mathematical methods for the calculation of LO and NLO. A possible improvement requires higher orders for the evaluation of the total cross section. Such terms are described as Next-to-Next-to-Leading order (NNLO). However has been show despite the non-perturbative nature of the proton, prediction for hadron colliders are feasible.