Time evolution of a small system couple to a quantum bath

Author: Imtiaj Khokon

Supervisor: Dr. Andrew Ho

Abstract

This project examined the time evolution of a composite system consists of a small sub system coupled to a bath such as a spin with level up and down and a bath state consists of 100 states for the small and long time limit. At t = 0 we let the subsystems to interact with each other for long time such as t = 0 to 40 and the average probability of finding the system in spin up state is calculated for large and small energy differences h between the spin levels. Furthermore, the behaviour of the fluctuations for small time such as t = 0 to 2 and t = 0 to 5 for small and large values of h are examined. It has been observed as the value of h increases the average probability of spin up state ρ_{11} and decay rate g increase as well and it is found that for small value of h the change of probability and decay rate are very small compared to the large value of h. In this project all of the calculations are done and graphs are plotted by using mathematica.

Model

At $t < 0$, the initial state of the composite system is the product of the spin and bath states $|\Psi_0\rangle = |S1\rangle \otimes |B3\rangle$ where bath states are made of random number ranges from −1 to +1 and spin state is up in z direction.
At $t = 0$, the spin up and bath states interact each other and evolve in time according to the equation of motion with unitary operator such as $|\Psi(t)\rangle = e^{-iHt} |\Psi_o\rangle$ under the influence of total Hamiltonian H which is given as

$$H = HS + HB + Hint$$

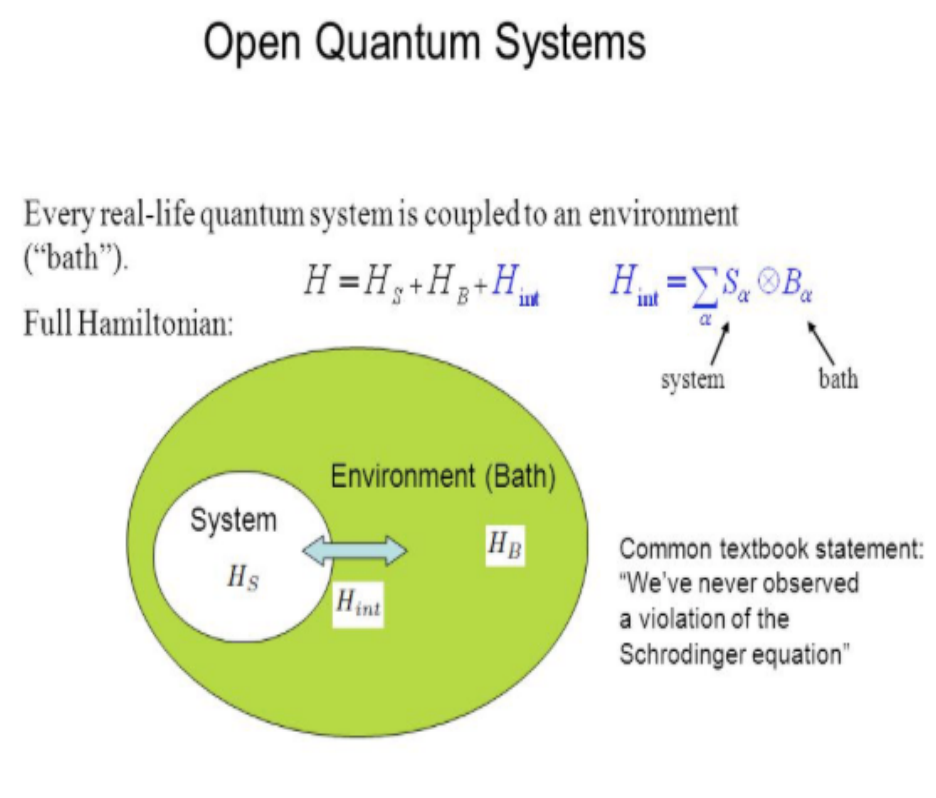

Figure 1: An open quantum system consisting of two subsystem, source: https://slideplayer.com/slide/4638629/

The total Hamiltonian needs to be Hermitian hence, the symmetry of the total Hamiltonian of the whole system is given as

$$H_{sm} = (H + Transpose[H])/2.$$

The probability of finding the spin up at long time is give as

$$\rho_{11}(t) = (1 - \rho_\infty) e^{-tg} + \rho_\infty .$$

Result

Table 1: The average probabilities and decay rates for large h values at long time limit.

h	Average probability ρ_{11}	Decay rate g
1	0.583573	6.31541
2	0.652639	8.91507
4	0.741896	17.7301
6	0.805068	174.014
8	0.850492	180.027
10	0.883767	180.19

Table 2: The average probabilities and decay rates for small h values at long time limit.

h	Average probability ρ_{11}	Decay rate g
0.01	0.502768	4.06119
0.03	0.503597	4.08102
0.05	0.504548	4.10413
0.07	0.505606	4.13011
0.09	0.506751	4.15845
0.1	0.507349	4.17332
0.5	0.537054	4.93365

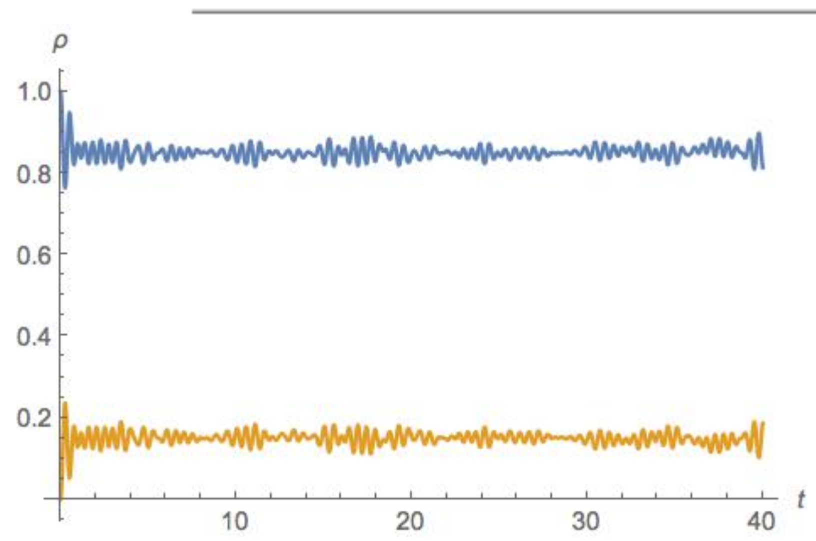

Figure 2: Long time behaviour of ρ at energy different h = 10 between spin levels.

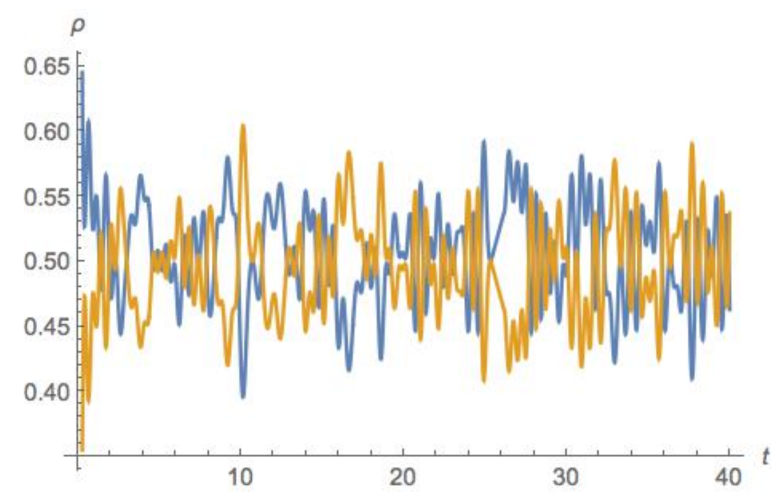

Figure 3: Long time behaviour of ρ when h = 0.01.

In Fig.(2 & 3) the upper curve represents spin up ρ_{11} and yellow down curve represents the spin down ρ_{22}. For large h value ρ_{11} and ρ_{22} do not intersect with each other whereas for small h value they do intersect.

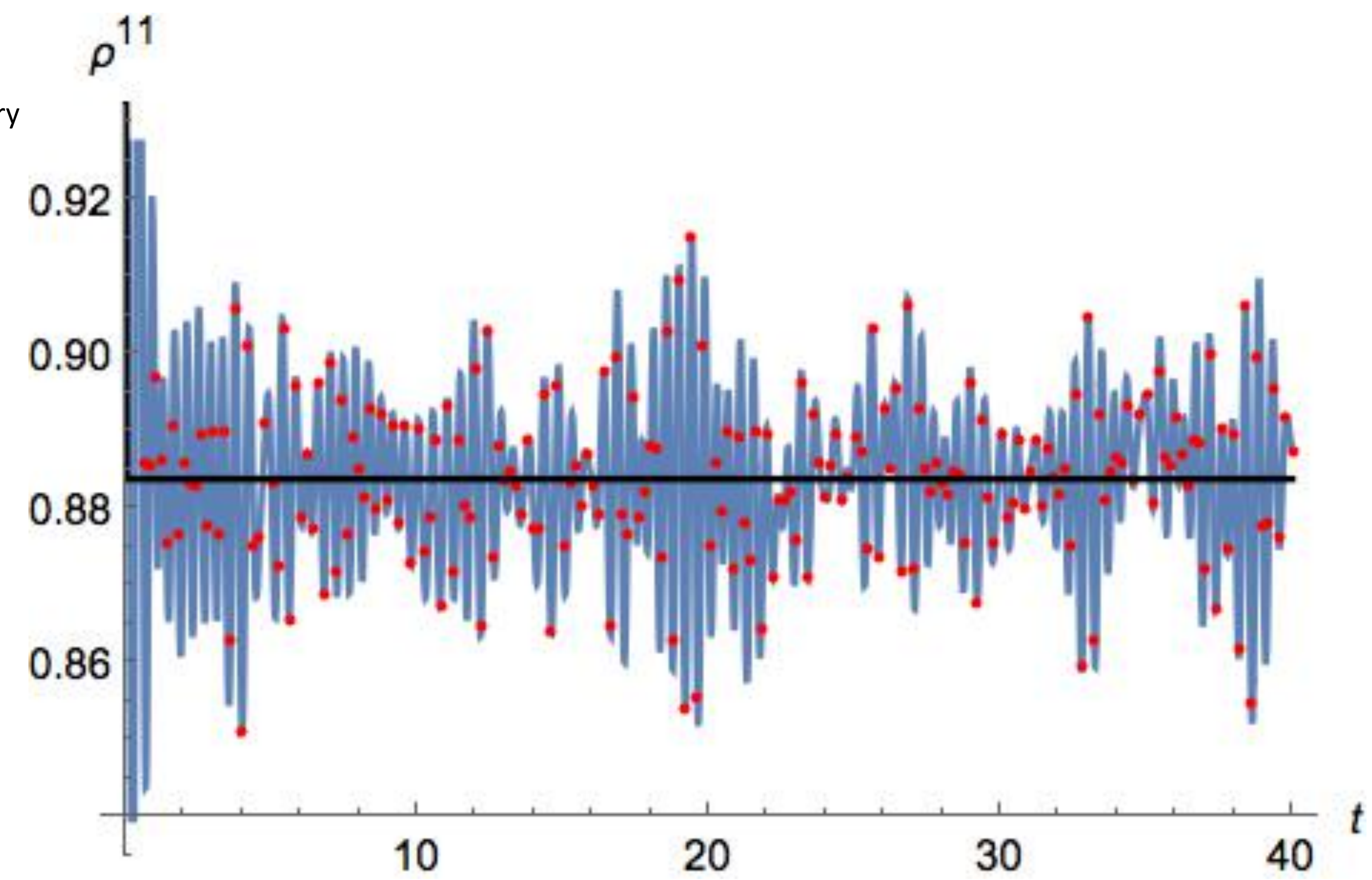

Figure 4: A graph of curve fitting for the long time behaviour of ρ_{11} for h = 10.

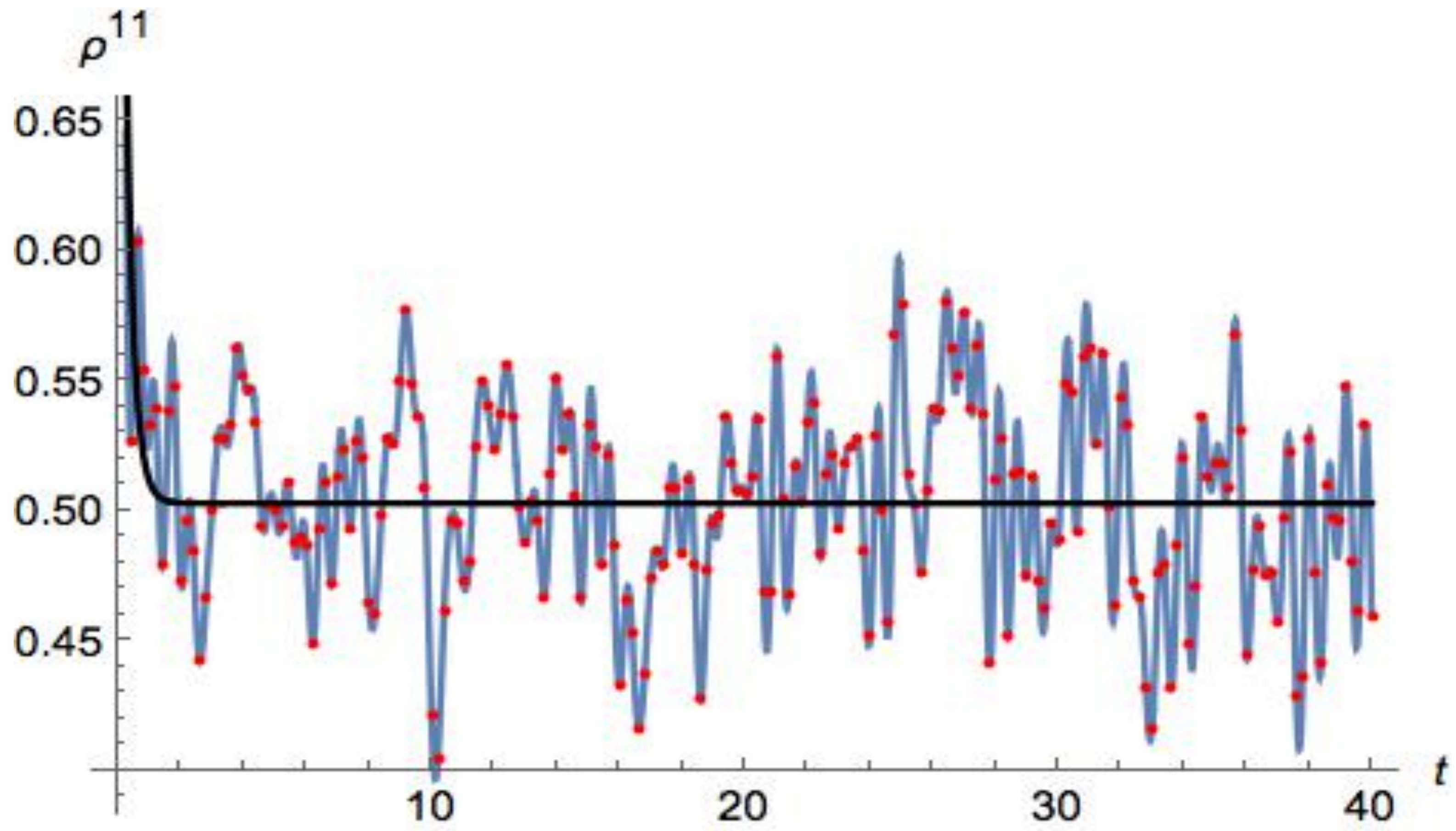

Figure 5: A graph of curve fitting for the long time behaviour of ρ_{11} for h = 0.01.

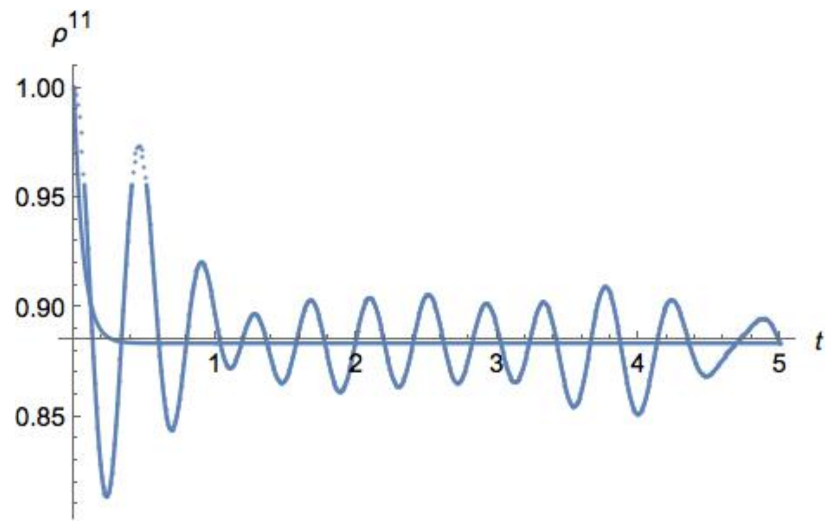

Figure 6: A graph of curve fitting for small time t = 0 to 5 for h = 10.

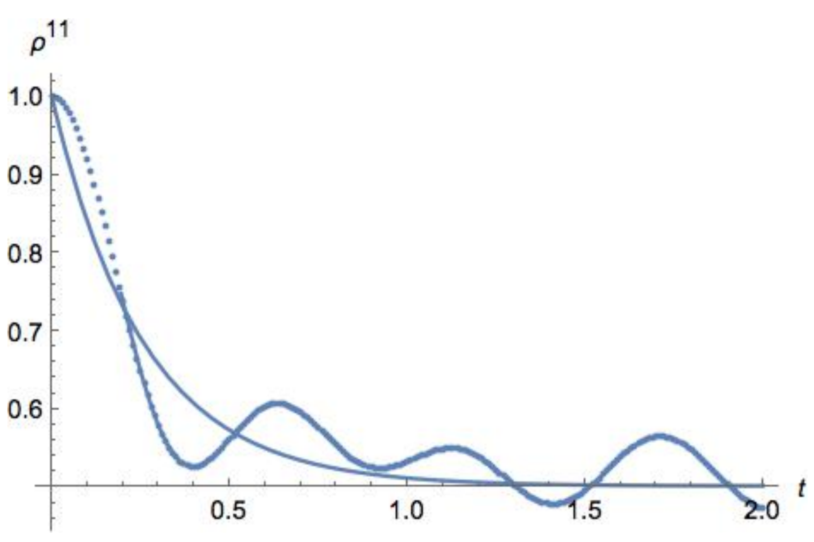

Figure 7: A graph of curve fitting for small time t = 0 to 2 for h = 0.01.

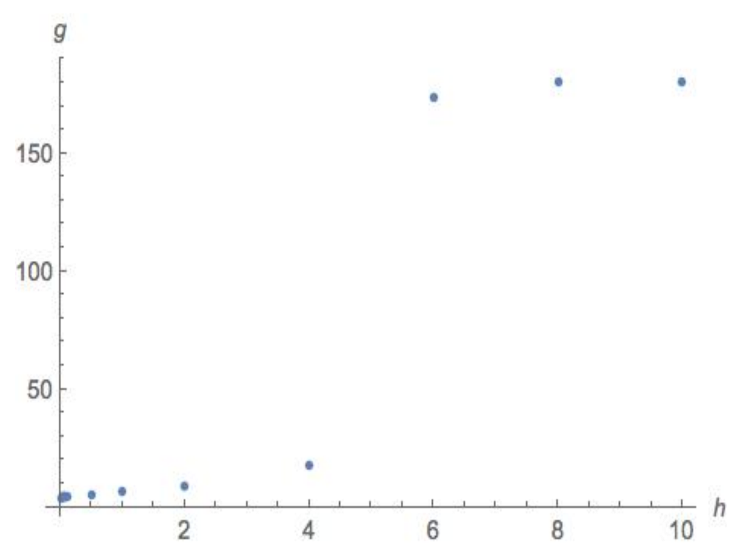

Figure 8: A graph of decay rate g vs energy difference h between spin levels.

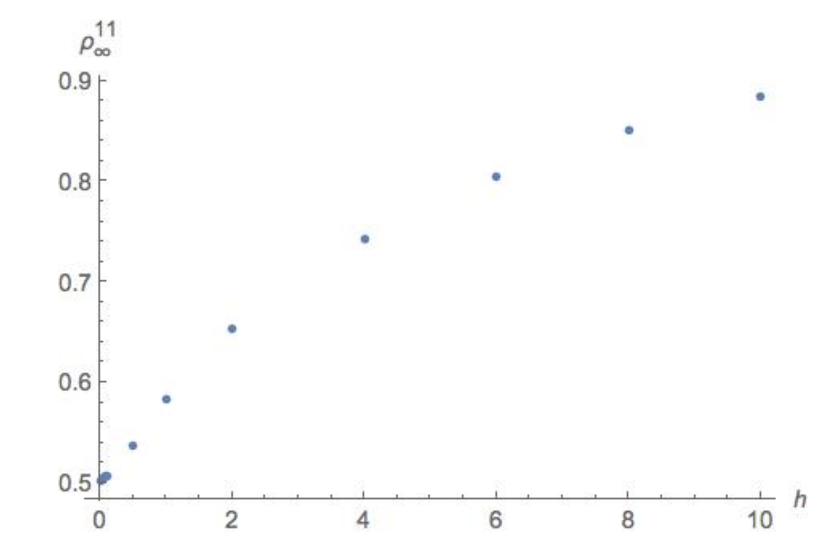

Figure 9: A graph of average probability ρ^{11} at long time vs energy difference h between spin levels.

It has been found that for the large energy difference such as h = 1 to 10, between spin levels the bath state can not help much to spin flip. In other words bath state does not have much influence over spin state due to the fact that bath sate consists of small energy ranges from −1 to +1. On the other hand, for the small energy difference such as h = 0.01 to 0.5, between the spin levels, bath state can help to spin flip. Hence, in this case the bath state can influence strongly the spin state.

Future work

The prospect for future studies of this project is to check the reliability of the data and fitting. One can do that by changing the range of time t or changing the step between t values. In addition one can also check the long time behaviour of the average probability ρ_{11} whether it decays as Boltzmann distribution or not against different energy differences h between the spin levels (see Fig.9).

ROYAL HOLLOWAY UNIVERSITY OF LONDON