Building a Dark Matter Model

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Abstract

By studying the observations that motivated its existence, the properties of dark matter are deduced. Dark matter interactions with Standard Model (SM) particles in the early universe are studied, from when they are in thermal equilibrium to when they freeze-out. This sets a constant value for amount of dark matter in the universe, and a relationship between this quantity and the relic abundance, Ωh^2. Then a Lagrangian for this interaction is constructed, along with a Feynman diagram from which the resulting matrix element is used to calculate the cross-section. This is used to find an expression for the annihilating cross-section that goes into the equation for Ωh^2. As this is a known cosmological parameter, we find a relationship between the mass of the dark matter particle χ , and its coupling strength to the Higgs.

Motivations

As dark matter cannot be detected directly, all the motivations for its existence came about through studying its effects on the observable universe through its interaction with gravity.

Motivations for the existence of dark matter include observations of the flatness of rotation curves of spiral galaxies, gravitational lensing, the velocities and luminosities of galaxies within a cluster, non-baryonic acoustic peaks in the anisotropies of the Cosmic Microwave Background or CMB (which tells us that dark matter is stable and non-baryonic), and simulations of the structure formation of the universe (which determined that dark matter is non-relativistic).

All of these phenomena cannot be explained by visible matter alone, they require the existence of a stable, non-baryonic, non-relativistic particle. In this model, the chosen candidate for a dark matter particle that fulfils these requirements is a Weakly Interacting Massive Particle, also known as the WIMP.

The Early Universe

In the era when all particles were in thermal equilibrium, dark matte particles interacted with SM particles in the following way

$$\chi\chi \rightleftharpoons \psi\bar{\psi}$$

As the universe expanded and cooled down, particles decoupled from the thermal plasma and this interaction froze-out. At this point the yield of DM particles is set, and is given by

$$Y_\infty = \frac{3.79(n+1)x_f^{(n+1)}}{\sqrt{g_*}M_{\text{Pl}}m_\chi\sigma_0}$$

The yield can be related to the relic abundance as follows

$$\Omega h^2 = 2.82\times10^8 Y_\infty m_\chi \text{GeV}^{-1}$$

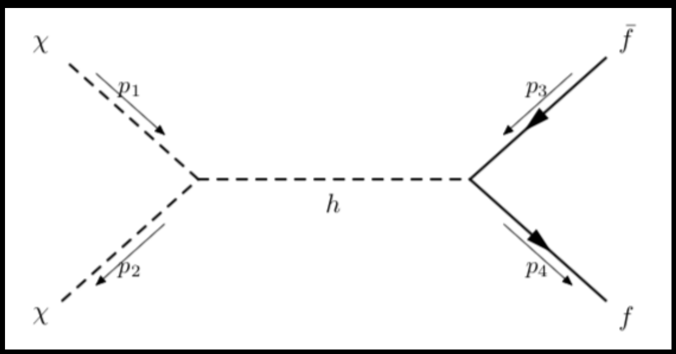
The relic abundance is a cosmological constant, its current value being 0.1198 ± 0.0012. Using this, we find values for the mass and coupling strength of this DM particle by finding an expression for the annihilation cross-section, σ_0 .

Building a Model

The dark matter particle χ is a scalar singlet. The Lagrangian for this dark matter model, given that χ transforms under a Z_2 symmetry, is

$$\mathscr{L} = m^2\chi^2 + \alpha\chi^2|H|^2 + \lambda\chi^4 + \mathscr{L}_{SM}\ , \text{ where } m_\chi = \sqrt{\mu^2 + \alpha v\left(\frac{v}{2}+h\right)}$$

The Feynman to the right displays an interaction between two DM particles, producing a final state fermion and anti-fermion pair, mediated by a Higgs boson. The differential cross-section is



$$d\sigma = \frac{1}{E_\chi^2|v_1-v_2|}|\mathscr{M}|^2\int\frac{d\Omega}{4\pi}\frac{1}{8\pi}\frac{2|p_f|}{\sqrt{s}}\ , \text{ where } \mathscr{M} = -iu(p_4)\lambda_f\bar{v}(p_3)\frac{1}{(s-m_h^2)}\lambda_\chi v$$

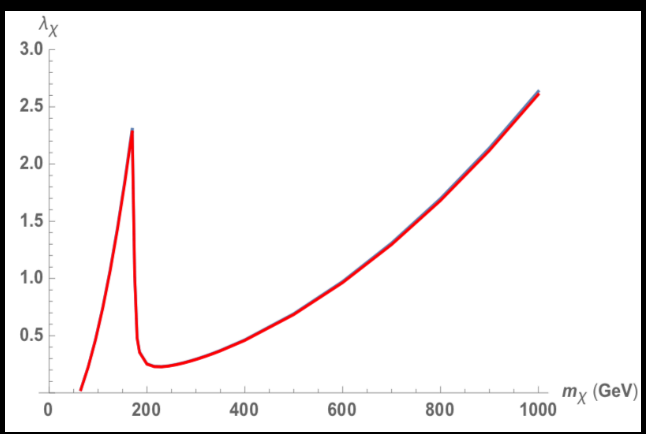
with $\mathscr{M}$ being the matrix element of this Feynman diagram.

Evaluating over the integral, substituting in the expression for the matrix element and finding everything in terms of the masses of particles and the coupling strengths, the final expression for the cross-section is obtained. This expression is then multiplied by the relative velocity, and a series expansion in terms of the relative velocity is found using Mathematica. The first term in the expansion is the annihilation cross-section.

$$\sigma = \frac{m_f^2\lambda_\chi^2}{4\pi m_\chi^2\gamma^2(\gamma^2-1)^{\frac{1}{2}}}\frac{(m_\chi^2\gamma^2-m_f^2)^{\frac{3}{2}}}{(4m_\chi^2\gamma^2-m_h^2)^2} \rightarrow \sigma v_r = \frac{m_f^2\lambda_\chi^2(m_\chi^2-m_f^2)^{\frac{3}{2}}}{2\pi m_\chi^3(m_h^2-4m_\chi^2)^2} - \frac{m_f^2\lambda_\chi^2(m_\chi^2-m_f^2)^{\frac{1}{2}}(m_f^2(28m_\chi^2-3m_h^2)-16m_\chi^4)}{16\pi(m_h^2m_\chi-4m_\chi^3)}v_r^2+\ldots \rightarrow \sigma_0 = \sum_f\frac{m_f^2\lambda_\chi^2(m_\chi^2-m_f^2)^{\frac{3}{2}}}{2\pi m_\chi^3(m_h^2-4m_\chi^2)^2}$$

This expression for the annihilation cross-section is put into the relic abundance equation, and the coupling strength of the dark matter to the Higgs is plotted over a range of mass values, for the upper and lower limit of the relic abundance.

Results and Analysis



The values obtained for the coupling strengths at different masses were plotted for bot the lower and the upper limit of the relic abundance. These separate plots were then superimposed onto one graph, the lower limit plot line in blue and the upper limit plot line in red. These plot lines are almost identical, which is indicative of the accuracy of the relic abundance. The lower limit for the mass is just over half the mass of the divergence in the cross-section. The general trend seen is the increase of the coupling strength to the Higgs with increasing dark matter mass. For the lower mass region, there is a steep increase in the coupling strength at around 175 GeV, the coupling strength decreases rapidly. At 173 GeV, the top quark contribution is added to the annihilation cross-section, as the two dark matter particles can now annihilate to produce a top and anti-top quark pair. As this large contribution has been added to the cross-section, the coupling strength would need to decrease significantly in order to maintain the yield and produce the correct relic abundance. Now that all the fermion contributions have been added, the coupling strength continues to increase with increasing mass.