VACUUM POLARISATION

Comparison of Methods for Loop Calculations

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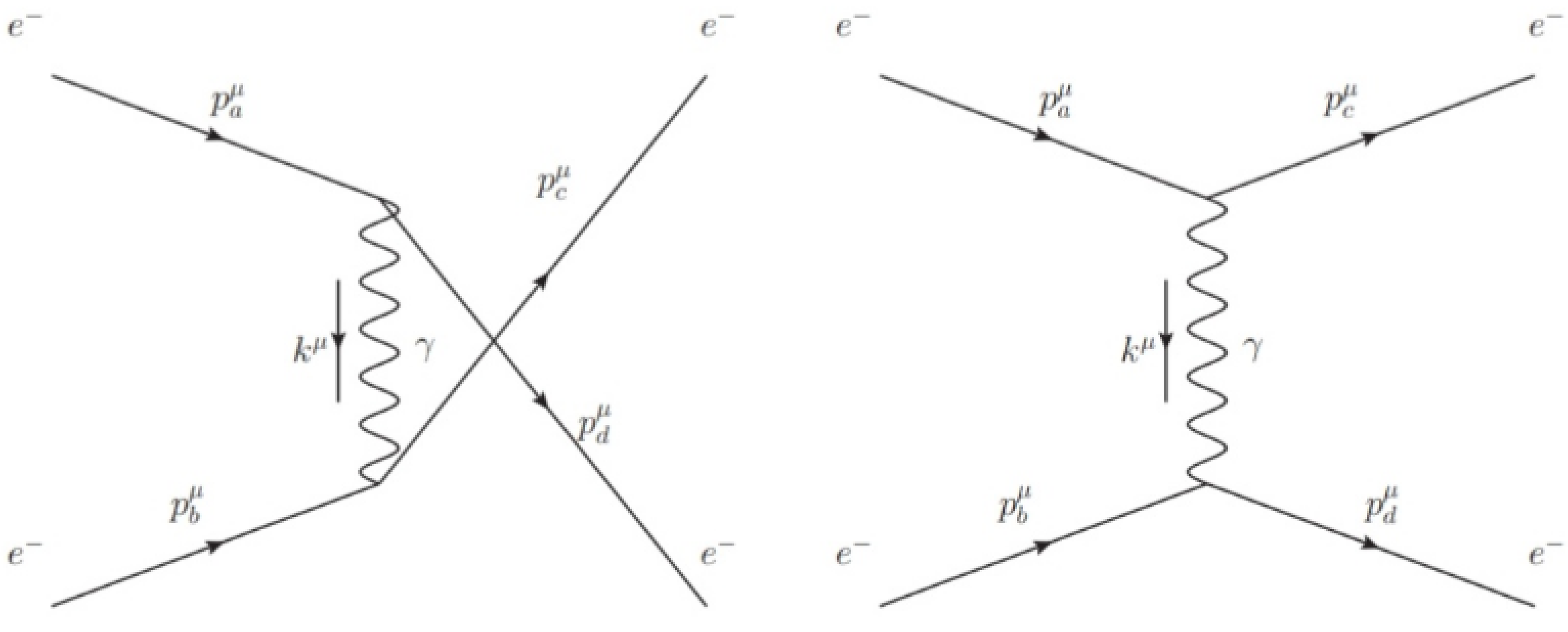
ABSTRACT: This project considered three different methods by which loop diagrams can be calculated: dimensional regularisation, unitary and generalised unitarity. The report also contained a discussion of calculating Feynman diagrams in general by considering Møller scattering (scattering of two electrons). An overview of the basics of renormalisation was also included.

FEYNMAN DIAGRAMS

Calculations in Quantum Field Theory (QFT) are simplified by perturbation theory. Each term in the perturbative series has a corresponding Feynman diagram[1]. These can be calculated with simple rules. For example, Møller scattering seen in Fig. (1).



(a) u-channel (b) t-channel

Fig.(1): Møller scatteting (the scattering of two elections) as an example of Feynman diagrams.

LOOP DIAGRAMS

Higher order diagrams can contain loops. The loop in Fig. (2) is known as the vacuum polarisation diagram[1].

This diagram is quadraticaly divergent (can analyse more carefully to make logarithmicly divergent with the ward identity).

Divergence is not seen in associated observables. Need to calculate with methods that separate the divergent component, then renormalise.

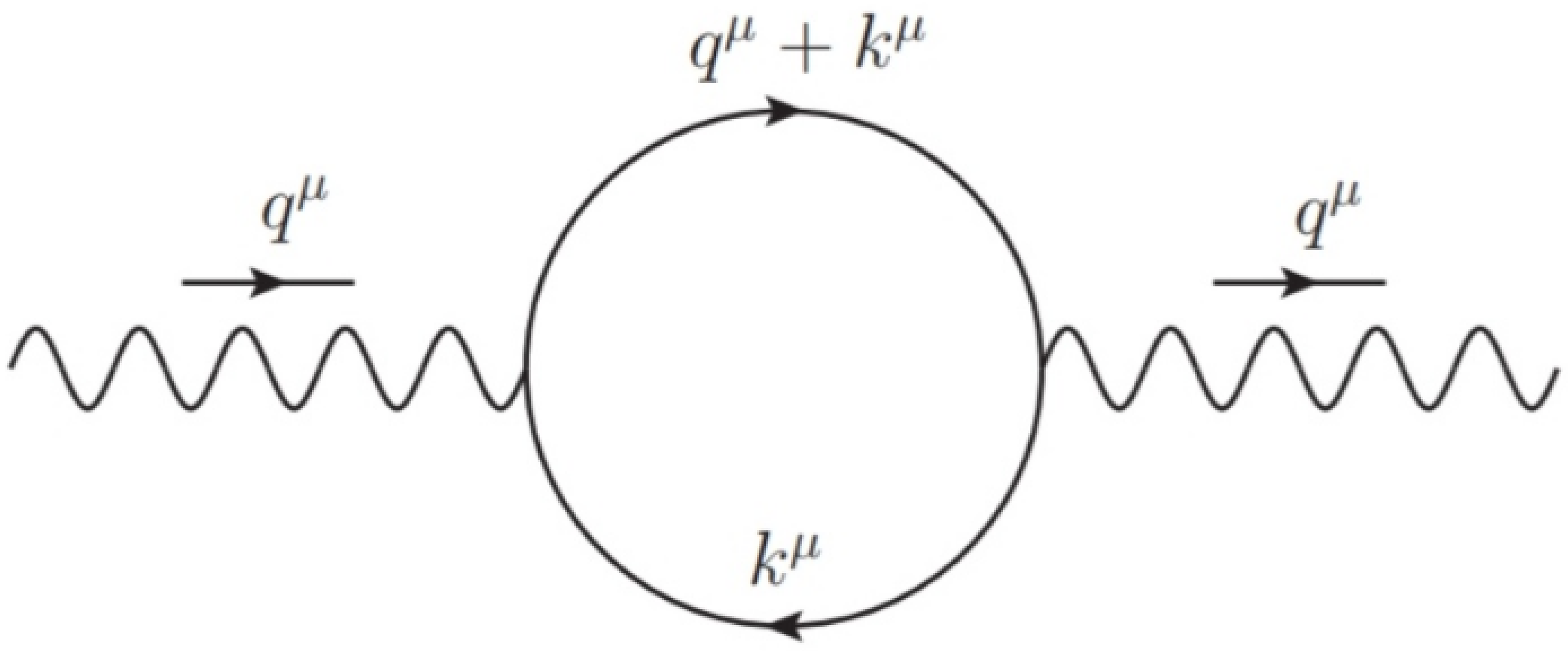
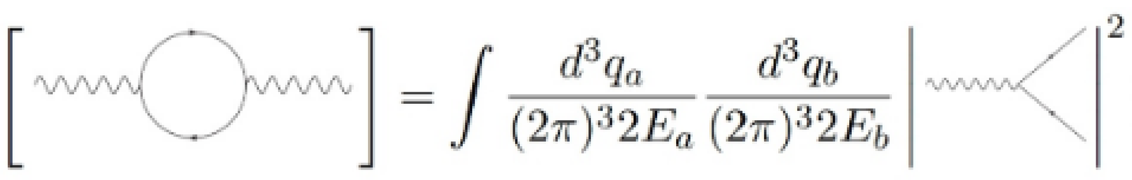


Fig.(2): The vacuum polarisation diagram. Can be considered as a loop correction to the photon propagator.

CALCULATION METHODS

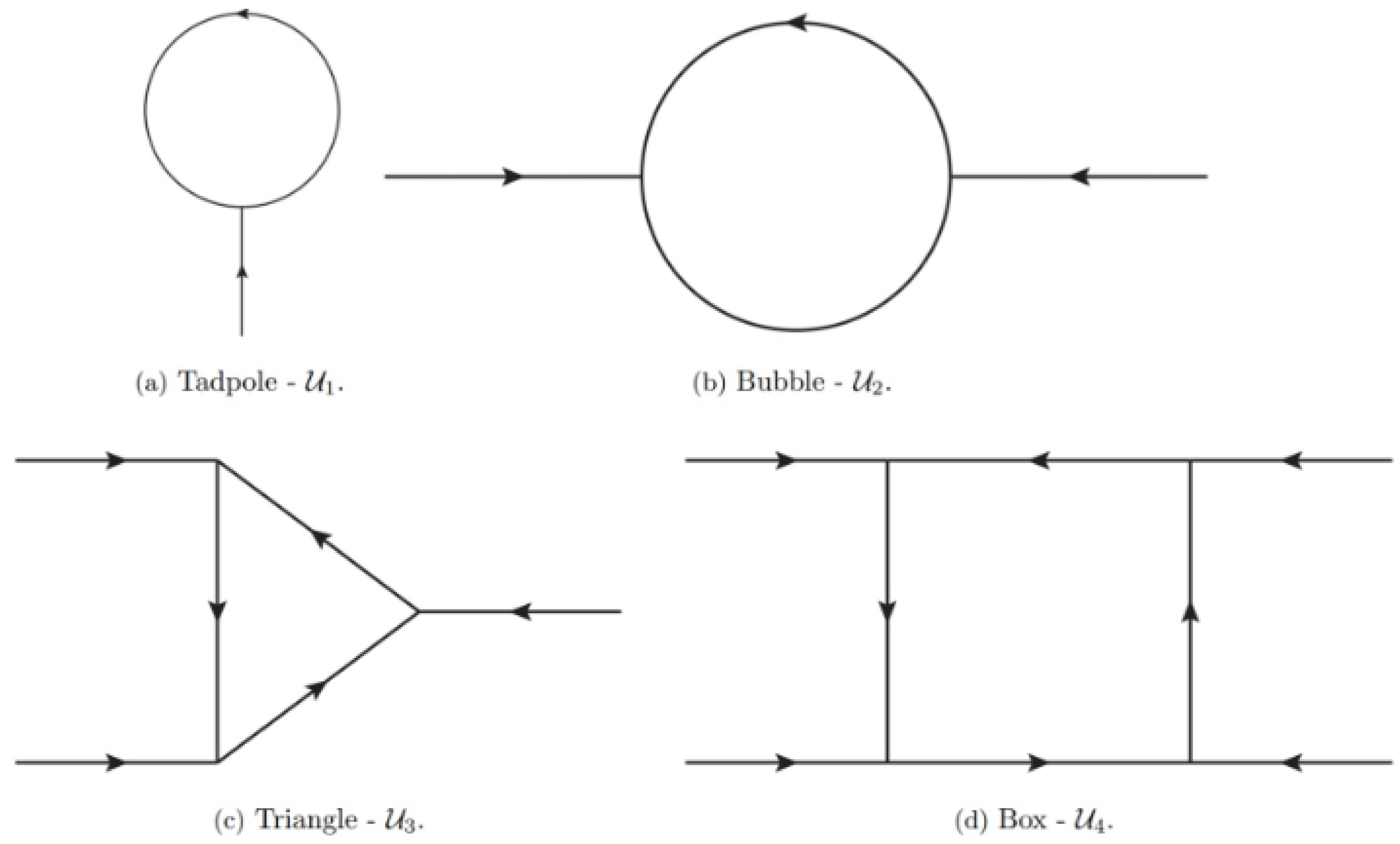
DIMENSIONAL REGULARISATION:
Generalise 4 momentum to n dimensions. Take limit of $n \rightarrow 4$ at the end of the calculation[1].

UNITARITY:
By noting the S-matrix is unitary, one can calculate imaginary part of loop from a pair production dia gram (half the loop diagram)[1]. Can then calculate the real part by using the Cauchy integral formula.



$$2\,\mathrm{Im}\left[\;\cdots\;\right] = \int \frac{d^3q_a}{(2\pi)^3 2E_a}\frac{d^3q_b}{(2\pi)^3 2E_b}\left|\;\cdots\;\right|^2 (2\pi)^4\delta^{(4)}(q_a^\mu + q_b^\mu - p_f^\mu),$$

GENERALISED UNITARITY:
Can always write a loop integral as a superposition of scalar loops, as seen in Fig. (3). As these are the same everytime, one only needs to find the coefficients.
One can find the coefficients by Cutkosky cutting rules. These replace propagators with dirac delta functions.



(a) Tadpole - $\mathcal{U}_1$. (b) Bubble - $\mathcal{U}_2$. (c) Triangle - $\mathcal{U}_3$. (d) Box - $\mathcal{U}_4$.

Fig.(3): The scalar loop diagrams that are used in the generalised unitarity method.

However this method leaves a extra component in the superposition, known as the rational term, undefined.

RENORMALISATION

The divergence still remains with these methods, it's just separated from the rest of the expression. Divergence is treated by allowing the physical charge of the electron e to differ from the lagrangian value e_0, and scale with energy. An overscore represents renormalised.

The Π, is the amplitude for all particle insertions to the photon propagator. Π_2 represents the second order correction (the vacuum polarisation diagram).

$$e^2(q^2) = \frac{e_0^2}{1 - \Pi(q^2)} \approx \frac{e_0^2}{1 - \overline{\Pi}_2(q^2)},$$

[1] M.E. Peskin and D.V. Schroeder. An Introduction To Quantum Field Theory. Frontiers in Physics. Avalon Publishing, 1995.